

## CFD modeling issues of wind turbine wakes under stable atmospheric conditions

K.G. Rados<sup>1</sup>, J.M. Prospathopoulos<sup>2</sup>, N.Ch. Stefanatos<sup>2</sup>,  
E.S. Politis<sup>2</sup>, P.K. Chaviaropoulos<sup>2</sup>, A. Zervos<sup>1</sup>

1: School of Mechanical Engineering, National Technical University of Athens

2: Centre of Renewable Energy Sources

e-mail: [kgrados@teikoz.gr](mailto:kgrados@teikoz.gr)

### Summary

In the present work, two Navier-Stokes solvers are applied to predict wake velocity deficits and turbulence intensity in the wake of a single wind turbine for different atmospheric stratification conditions. The two solvers use different turbulence closure, the k- $\epsilon$  and k- $\omega$  model respectively. Results from both solvers showed underestimation of the near wake velocity deficit. This can be justified by the presence of higher turbulence dissipation rate caused by the wind turbine rotor and/or the atmospheric stability. Three different modeling approaches are examined to simulate this mechanism: Addition of a turbulence dissipation production term in the wind turbine surrounding area, modification of the turbulence model constants for stable atmospheric conditions, and direct change of the turbulence length scale. Comparison of predictions with full-scale and wind tunnel measurements showed that the different approaches can give satisfactory results regarding velocity deficit and turbulence intensity. However, further validation with experimental data is required to better understand the physical mechanism of the interaction a wind turbine wake with the atmospheric boundary layer.

**Key words:** Wind turbine wake, atmospheric stability, CFD model

### 1. Introduction

Wind turbine (WT) wake modeling has attracted a lot of attention by the wind energy research community since the flow in the wake is characterized by momentum deficits and increased turbulence levels affecting the performance of downstream turbines by reducing their power output and increasing the fluctuating loads. As wakes develop downstream, they interact with atmospheric boundary layer as well as with other wakes. The size and evolution of wake structures depends on many factors such as the ambient wind speed and turbulence, wake added turbulence, the turbine characteristics, the terrain and the structure of the boundary-layer relating to atmospheric stratification. Accurate prediction of wind velocity deficit downstream of the W/Ts is crucial to the estimation of the power output and loading of a wind farm.

Development of a wind farm model requires detailed understanding of flow and the interaction of complex atmospheric structures with those generated by W/Ts. In some cases, wind farms may operate in stably stratified atmosphere, especially during night hours. The need for reliable wake modeling has been recognized for many years and various models have been developed to simulate W/T wakes. Most of wake and wind farm models were developed in the 1980's for small wind farms. Many of them were based on the work of [1], [2] and [3]. By necessity, wake models had to be fast (low computational cost) implementing explicit analytical expressions of the wind farm efficiency as a function of layout, atmospheric turbulence and turbine characteristics. For single wakes or small wind farms those models provided results in satisfactory agreement with available data [4], [5]. However, they are not capable of properly taking into account near wake development and atmospheric stability effects. The substantial progress in computer capabilities has permitted cost efficient calculations using more sophisticated methods such as the solution of Navier-Stokes equations in their RANS approximation with appropriate turbulence modeling (CFD models). Wake modeling can no longer be based on engineering approximations and the perspective is to perform CFD simulations even for large wind farms in complex terrain or offshore [6]. Those sophisticated models can provide detailed spatial variation of the wind field within the wind farm by treating the generation, development and interaction of wakes with the atmospheric boundary layer as a whole using the CFD approach.

In the present work, two CFD Navier-Stokes solvers called 3D-NS [4], [7] and FlowNS [8] are applied and compared for the single wake of a wind turbine under different atmospheric stratification conditions. The main difference between 3D-NS and FlowNS solvers is the use of different turbulence closure schemes the k- $\epsilon$  and k- $\omega$  model respectively. Initial calculations of both solvers showed underestimation of the near wake velocity deficit, especially at the nearest measuring position. This can be justified by the presence of higher turbulence dissipation rate caused by the W/T turbine and/or the atmospheric stability. Different aspects of modeling are examined to simulate this mechanism: addition of a turbulence dissipation production term in the W/T surrounding area, modification of the turbulence model constants for stable atmospheric conditions, and finally direct change of the turbulence length scale. Comparison with measurements shows the effect of the above modeling considerations on the velocity deficit and turbulence profiles.

## 2. The CFD models

Both CFD models solve numerically the 3D Reynolds averaged incompressible Navier-Stokes equations (RANS) on a Cartesian grid. W/Ts are accommodated in their grid as momentum sinks (pressure jumps) representing the axial force applied on the rotor disk that is in turn evaluated from the given thrust coefficient curve. The governing equations are numerically integrated by means of an implicit pressure correction scheme. A matrix-free algorithm for pressure updating is introduced, which maintains the compatibility of the velocity and pressure field corrections, allowing for practical unlimited large time steps within the time integration process. Spatial discretization is performed on a computational domain, resulting from a body-fitted coordinate transformation, using finite difference/finite volume techniques. The convection terms in the momentum equations are handled by a second order upwind scheme bounded through a limiter. Centred second order schemes are employed for the discretization of the diffusion terms. The Cartesian velocity components are stored at grid-nodes while pressure is computed at mid-cells. This staggering technique allows for pressure field computation without any explicit need of pressure boundary conditions. A linear fourth order dissipation term is added into the continuity equation to prevent the velocity-pressure decoupling.

Turbulence closure is achieved using the standard k- $\epsilon$  (in 3D-NS) [9], and k- $\omega$  (in FlowNS) [10] models, appropriately modified for atmospheric flows [11] More specifically the following constants are used:

$$\text{3D-NS:} \quad C_{\epsilon 1}=1.12, C_{\epsilon 2}=1.83, C_{\mu}=0.033, \sigma_k=1, \sigma_{\epsilon}=1.3$$

$$\text{FlowNS:} \quad \alpha=0.3706, \beta=0.0275, \beta^*=0.033, \sigma=0.5, \sigma^*=0.5$$

In order to deal with stable atmospheric stratification and using the similarity theory, a buoyancy production term  $G$  was added in the k equation throughout the whole computational domain [12], [13]:

$$G = -\mu_t \left( \frac{\partial U}{\partial z} \right)^2 \cdot \frac{Ri}{f_m}, \quad Ri = \zeta \frac{0.74 + 4.7\zeta}{(1 + 4.7\zeta)^2}, \quad f_m = 1 + 5\zeta, \quad \zeta = z / L_m$$

where  $\mu_t$  is the turbulent viscosity,  $Ri$  is the Richardson number and  $L_m$  is the Monin-Obukhov length.

In both horizontal directions, the grid size is constant and equal to  $0.05D$  close to the W/T ( $-0.55D$  to  $0.55D$ ), and increases outwards, following a geometrical progression, until the maximum computational domain size is reached. In the vertical direction, the first three grid-lines are positioned close to the ground at heights  $0.01$ ,  $0.03$  and  $0.05D$  respectively. In this way, a fine mesh is constructed in the area of the W/T rotor disk with 21 grid points across its diameter. The presence of the W/T is introduced by adding a sink term in the axial velocity momentum equation using the actuator disk force calculated through the thrust coefficient.

The inflow wind velocity profile for stable conditions follows the logarithmic law [12], [13] (for neutral conditions  $\Psi_m=0$ ):

$$U_x = \frac{U_*}{\kappa} [\ln(z/z_0) - \Psi_m], \quad \Psi_m = -5z/L_m$$

where  $z_0$  is the roughness length and  $\kappa$  the Von Karmann constant. The inflow profiles of  $k$  and  $\varepsilon$  (or  $\omega$ ) are determined using the following expressions (for neutral conditions  $f_\varepsilon = f_m = 1$ ):

$$k = u_*^2 / \sqrt{\beta_*} (f_\varepsilon / f_m)^{0.5}, \quad \omega = u_* / (\sqrt{\beta_*} K z) (f_\varepsilon f_m)^{0.5}, \quad \varepsilon = u_* C_\mu / (\sqrt{\beta_*} z) (f_\varepsilon f_m)^{0.5}$$

### 3. Results

The following different assumptions in turbulence model are considered and studied to approximate the enhanced turbulence dissipation mechanism causing the delay in the velocity deficit attenuation observed especially in the near wake:

- An additional turbulence dissipation rate production term was incorporated in the epsilon and omega equations for the cylindrical volume surrounding the wind turbine rotor [14]. According to this assumption the presence of the turbine results in canceling the local turbulence equilibrium in this area. The extent of the surrounding volume was taken as 0.25 diameters upstream and downstream of the rotor as suggested by [14]. The additional production term is multiplied by a constant  $C$  which can be calibrated to fit experimental data. In the  $k$ - $\varepsilon$  model, this constant was tuned to 0.37, whereas in the  $k$ - $\omega$  model the relative constant was tuned to 4.

$$P_\varepsilon = C \cdot P_t^2 / \rho k, k - \varepsilon$$

$$P_\omega = C \cdot P_t^2 / \rho k^2, k - \omega$$

where  $P_t$  is the turbulence production term in the  $k$  equation.

- The constants of the standard  $k$ - $\varepsilon$  and  $k$ - $\omega$  models were modified. For neutral atmospheric conditions the decay turbulent ratio  $\beta_* / \beta = 1 / (C_{\varepsilon 2} - 1)$  was reduced (initial value is 1.2). For stable atmospheric conditions the constants were modified according to [15].

- The length scale  $L$  of turbulence was decreased and used to determine the initial turbulence dissipation rate profile through its expression with respect to  $k$ :  $\varepsilon = k^{1.5}/L$

In the sequel, model results using the above approximations are evaluated through comparisons with measurements for three single wake cases.

#### 3.1 Perforated disk in wind tunnel flow [16]

The first test case was the simulation of an experiment [16] undertaken in the closed loop type wind tunnel of NTUA. The testing area had a cross section of 8.75 m<sup>2</sup> (3.5m x 2.5m) and a length of 11.5m. The W/T was simulated by a non-rotating plastic perforated disk 8mm thick with 200mm diameter. The thrust coefficient of the simulator was estimated equal to 0.8. In the present simulation, the free stream velocity was 10m/s and the ambient turbulence intensity 10%. Since a wind tunnel flow is simulated, the standard  $k$ - $\varepsilon$  model constants were used.

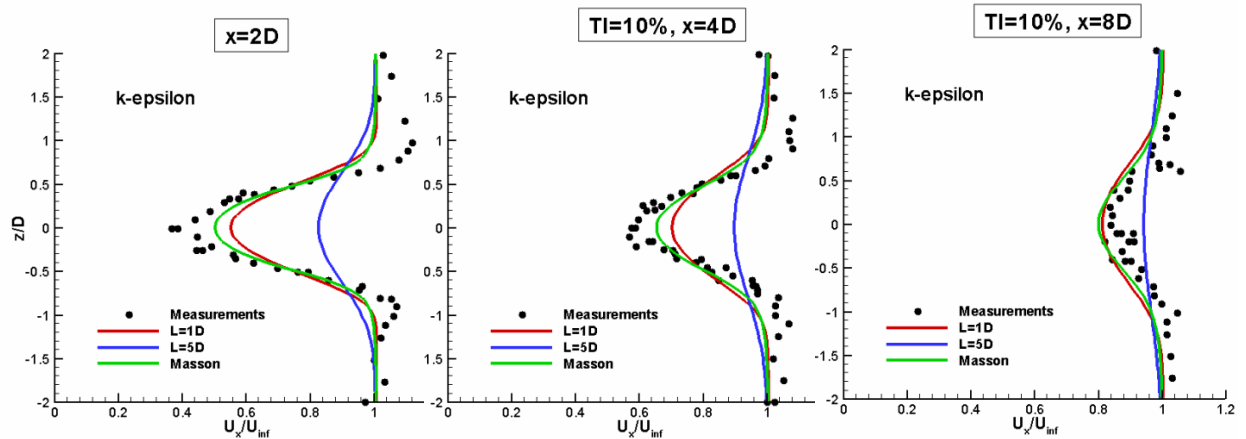


Figure 1. Normalized axial velocity profiles in the wake of a perforated disk in a wind tunnel at 2D, 4D and 8D downstream of the disk using different turbulence modeling approaches

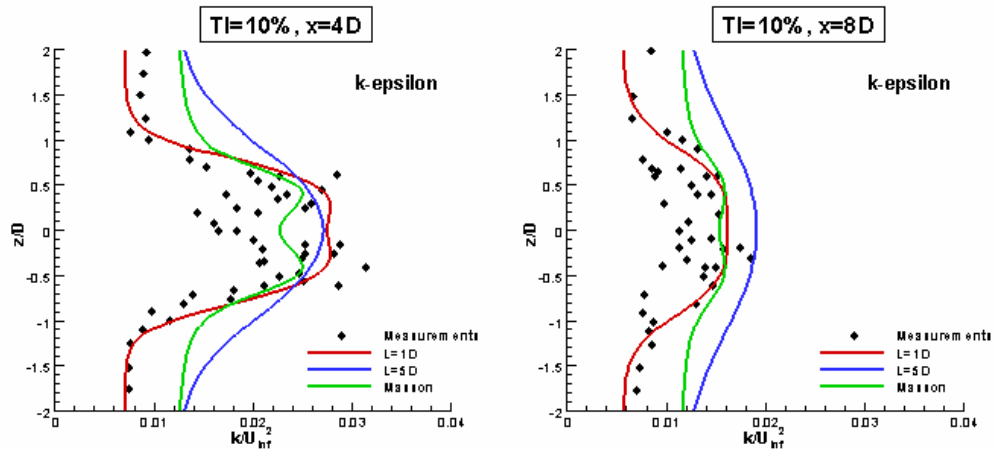


Figure 2. Normalized turbulence kinetic energy profiles in the wake of a perforated disk in a wind tunnel at 4D and 8D downstream of the disk using different turbulence modeling approaches

3D-NS model results are compared with wind tunnel measurements in Figures 1 and 2. Figure 1 present velocity profiles at distances 2, 4 and 8 diameters whereas turbulence kinetic energy profiles are given at two downstream positions 4 and 8 diameters downstream of the disk. Clearly, the length scale  $L$  value at the inlet plane of the computational domain affects dramatically the numerical predictions. Comparison against the measurements suggests the use of disk diameter rather than the wind tunnel scale to determine the initial dissipation rate  $\varepsilon=k^{1.5}/L$ . Introduction of the correction in turbulence modeling proposed in [14] and using  $L=5D$  seems to perform well regarding wake velocity deficits and turbulence in the wake but overestimates turbulence outside the wake as observed also for the standard  $k-\varepsilon$  model with  $L=5D$ .

### 3.2 Experiment on the Nibe wind turbines [17]

The second test case was the measuring campaign undertaken in 1990 in Denmark for the two 630kW Nibe W/Ts with 40m diameter and 45m hub height, separated by a distance of 5 diameters. The velocity deficits and turbulence intensities were measured up to a distance of 7.5 diameters. In the present simulation, the wind direction aligned with the W/T and the measuring mast location was 188 degrees, the free stream velocity was 8.55 m/s and the

ambient turbulence intensity was 9%. The thrust coefficient was estimated equal to 0.82. Neutral atmospheric conditions were dominant during the experiment.

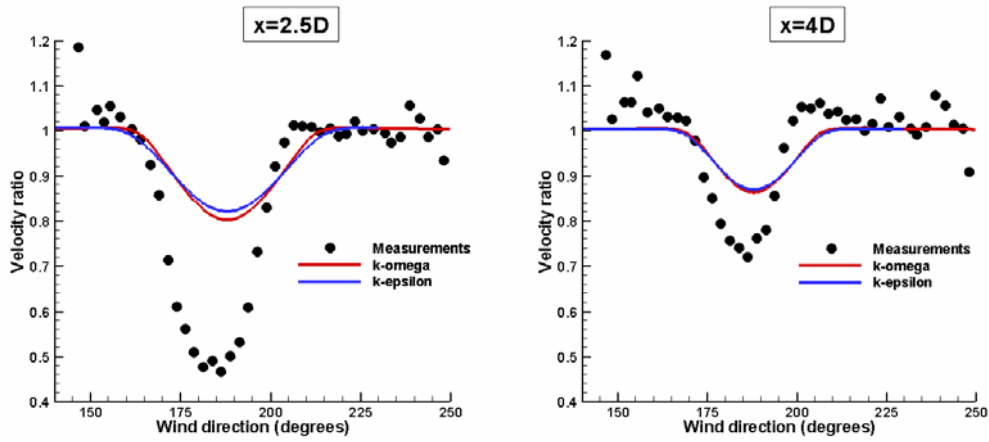


Figure 3. Wake velocity ratio with respect to wind direction using the standard k- $\epsilon$  and k- $\omega$  constants for the Nibe W/Ts at distances 2.5D and 4D downstream of the rotor plane

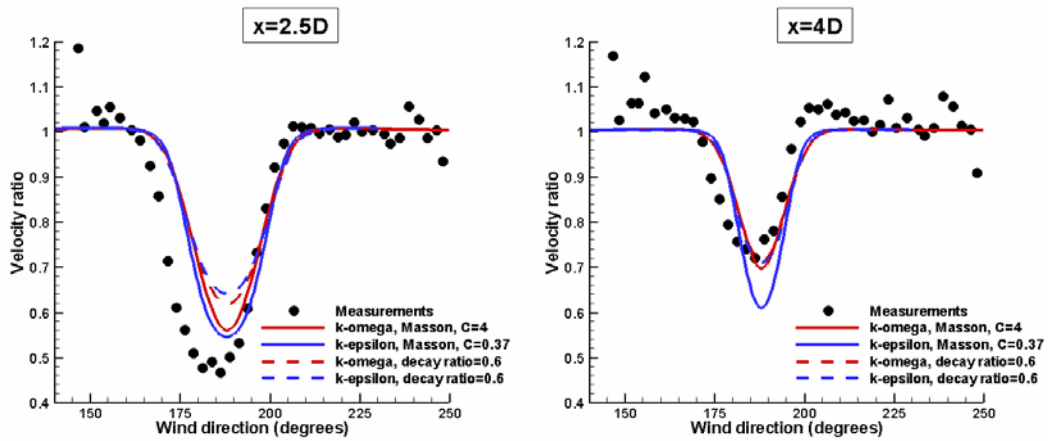


Figure 4. As in Figure 3 but using different turbulence modeling approaches

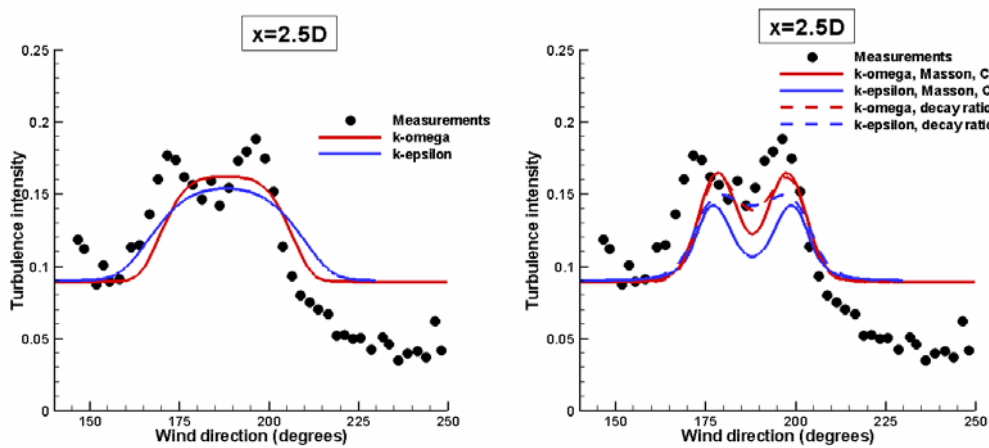


Figure 5. Turbulence intensity with respect to wind direction using the standard k- $\epsilon$  and k- $\omega$  constants and different turbulence modeling approaches for the Nibe W/Ts

Numerical predictions are compared with measurements in Figures 3, 4 and 5. Although they have almost identical behavior, a significant overestimation in the wake velocities is observed in Figure 3 from both models when using the standard turbulence modeling constants. However, predictions presented in Figure 4 are improved by introducing enhanced dissipation rate in the W/T area proposed in [14] or equivalently changing the turbulence decay ratio  $\beta^*/\beta$ .

Better performance from both models in turbulence predictions is also observed in Figure 5 when the above corrections are incorporated. The “double peak” pattern in the near wake (2.5D) is quite well reproduced by the models while the standard k- $\epsilon$  and k- $\omega$  results fail to predict it.

### 3.3 ECN full-scale experiment [18]

The third test case refers to the measuring campaign at the EWTW test wind farm of ECN consisted of five 2.5 MW W/Ts with 80m diameter and 80m hub height. The distance between the W/Ts is 3.8 diameters. In the present simulations of a single W/T the prevailing wind direction was 45 degrees the free stream wind velocity was 9 and 11 m/s and the ambient turbulent intensity was 11.7%. The thrust coefficient was 0.76 and 0.63 for wind velocities 9 and 11m/s respectively.

The turbine operated in stably atmospheric conditions and hence the buoyancy generation term was added in the turbulence kinetic energy equation. Velocity profiles predicted by FlowNS with k- $\omega$  are compared with measurements in Figure 6. Four different approaches are examined:

Standard k- $\omega$  – *no correction*

Different constants proposed in [15] – *Freedman’s model*

Correction proposed in [14] – *Masson C=4*

Use of a constant inlet length scale  $L$  equal to 0.1D to determine  $\omega$  instead of using its atmospheric boundary layer profile – *Scale L=0.1D*

Evidently, the standard k- $\omega$  model performs better for stable stratification than in neutral as seen for the Nibe W/T in 3.2. The predictions are further improved by increasing  $\epsilon$  in the W/T area or decreasing the length scale  $L$  or change constants of k- $\omega$  model. All three alternative approaches are equivalent their effect being to increase the turbulence dissipation rate resulting in decrease of the turbulence kinetic energy, enhancement of the turbulence dissipation mechanism and hence delaying the velocity deficit attenuation in the wake.

## 4. Conclusions

Two Navier-Stokes solvers with k- $\epsilon$ , k- $\omega$  turbulence models were applied to simulate wake effects of a single wind turbine. Standard turbulence modelling schemes and constants underestimate wake effects. Larger discrepancy of model results with available wind tunnel and full scale measurements is observed for neutral atmospheric conditions. In both neutral and stable atmospheric conditions, predictions are improved by increasing the turbulent dissipation rate close to the wind turbine, using a smaller turbulent length scale or changing the turbulence parameters so that the decay ratio is decreased. There is an indication that near wake turbulence is associated with a smaller length scale in the wind turbine region. Comparison of predictions with measurements showed that the different approaches can give satisfactory results regarding velocity deficit and turbulence intensity. However, further validation with experimental data is required to better understand the physical mechanism of the interaction of a wind turbine wake (small scale turbulence) with the atmospheric boundary layer (large scale turbulence).

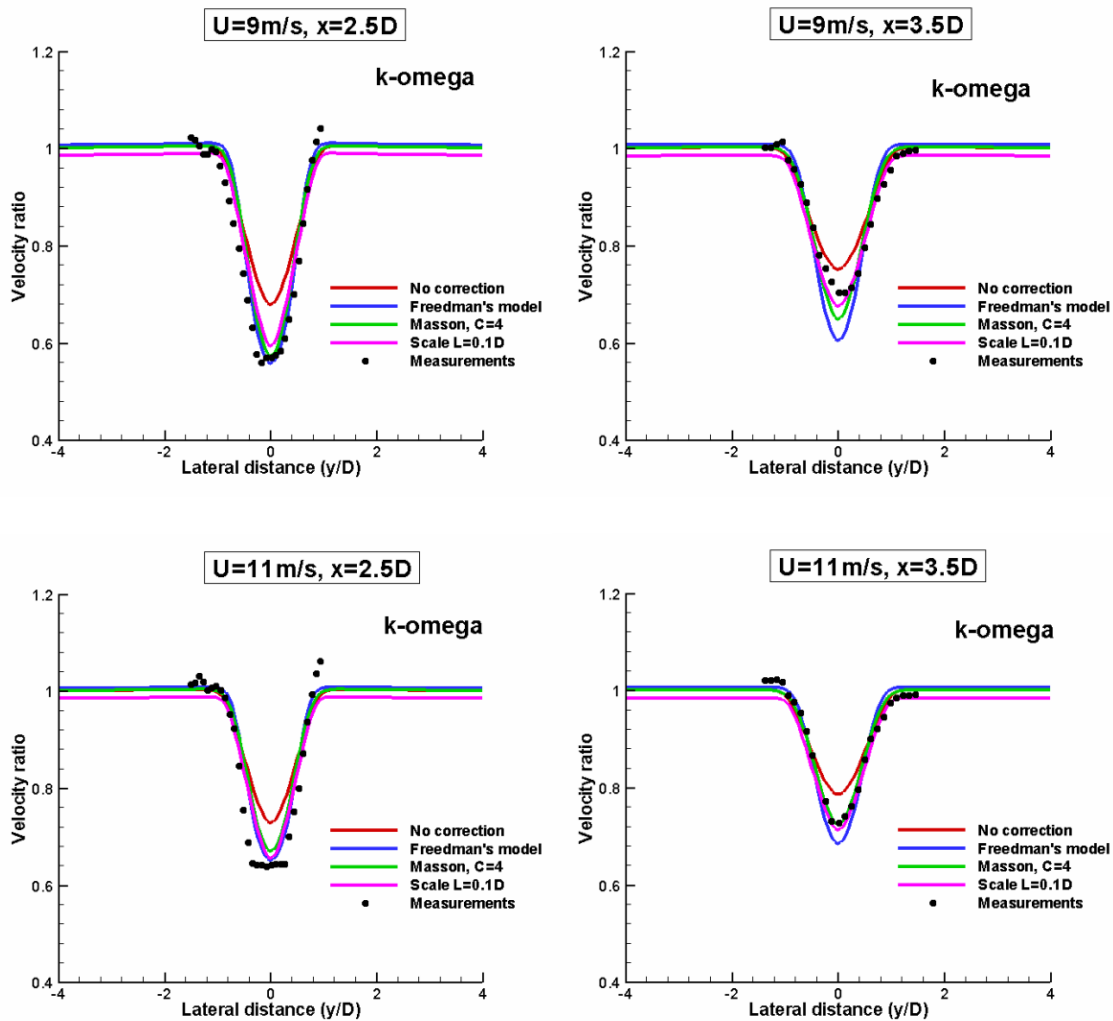


Figure 6. Normalized axial velocity profiles in the wake of the 2.5MW W/T for 9 and 11 m/s free-stream wind speed and at distances of 2.5D and 3.5D downstream of the rotor disk using different turbulence modeling approaches

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