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Modeling of fatigue damage evolution on the basis of the kinetic concept of strength

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Abstract On the basis of the kinetic theory of strength, a new approach to the modeling of material degradation in cyclic loading has been suggested. Assuming that not stress changes, but acting stresses cause the damage growth in materials under fatigue conditions, we applied the kinetic theory of strength to model the material degradation. The damage growth per cycle, the effect of the loading frequency on the lifetime and on the stiffness reduction in composites were determined analytically. It has been shown that the number of cycles to failure increases almost linearly and the damage growth per cycle decreases with increasing the loading frequency.

Keywords Kinetic theory of strength · Fatigue · Damage · Lifetime · Frequency

1 Introduction

One of the oldest problems in the fatigue analysis has been the analysis of the interrelations between the loading conditions and the lifetime of materials (number of the cycles to failure) (Suresh 1998; Palmgren 1924; Miner 1945; Paris et al. 1961; Fatemi and Yang 1998). Among the analytical, experimental and statistical approaches, used to investigate this problem, one may list the concepts based on the Wöhler curve and Basquin equation, rain-flow counting, Palmgren-Miner's rule of damage accumulation, Paris law and different generalizations of the fracture and damage mechanics approaches The main challenge in most of these works was to take into account explicitly the temporal effects in fatigue, which do influence the lifetime of materials (Suresh 1998). However, many of the underlying concepts and approaches used either do not take into account the temporal effects (as fracture mechanics), or are based on the data analysis and averaging (as Wöhler curves and Paris law) (Fatemi and Yang 1998).

In particular, the problem of the effect of loading frequency on the damage growth and lifetime of materials can be hardly explained in the framework of the static concepts. As noted by Parsons et al. (2000), "for different polymers, the crack growth rate (expressed in units of length per number of cycles) may decrease, remain nearly constant, or increase with increasing frequency". Hertzberg et al. (1975) studied the effect of test frequency on polymer fatigue performance, seeking to explain a diminution of fatigue resistance with increasing cyclic frequency in unnotched test samples, and the enhancement of fatigue resistance in many polymers with increasing cyclic frequency

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in notched samples. As noted by Hertzberg et al. (1975), contradictory trends in frequency-sensitive materials properties are responsible for these differences. The relative fatigue behavior reflects "a competition between strain rate and creep effects", as well as the effect of β transition in polymers (Hertzberg et al. 1975).

Takemori (1992) noted that the conclusions on the frequency effects made on the basis of an analysis of unnotched specimens are not transferable on notched specimens case. Moskala (1993) also noted that the fatigue resistance of the untoughened amorphous blend of polycarbonate was not affected by test frequency, whereas the fatigue resistance of the toughened blend increased with increasing frequency. Mandell and Meier (1983) studied load frequency effects for cross-ply E-glass/epoxy laminates, carrying out tests with three frequencies (0.01, 0.1, and 1 Hz), and observed that the number of cycles increased with increasing load frequency.

Saff, and other researchers also considered the effect of frequency on the fatigue behavior (Saff 1983; Sun and Chan 1979; Rotem 1993). These and other results were summarized in Hahn and Turkgenc (2000) as follows: "at low frequency ranges where there is negligible heat dissipation, as the load frequency increases, cycles to failure increase also. As higher frequency ranges are considered this increase is at a slower rate. When there is excessive heat dissipation, however, a reverse trend can be observed."

One of the ways to analyze the time-dependent effects on the fatigue crack growth is to consider the crack growth rate as a superposition of fatigue and creep components (Hertzberg et al. 1975; Wnuk 1974). So, Lee et al. (2003) analyzed the damage growth in polymer composite materials on the basis of the fracture mechanics model by Wnuk (1974). Using the transition from fracture to damage mechanics concept, they derived the following formula

$$\mathrm{dD/dN} = \mathrm{c}_1(\sigma^2_{\mathrm{max}}/\mathrm{D})^{\mathrm{m}} + (\mathrm{c}_2/\mathrm{f})(\sigma^2_{\mathrm{max}}/\mathrm{D})^{\mathrm{n}} \quad (1)$$

where c_1, c_2, m, n are the parameters of the material.

The interrelations between the static and fatigue failure was considered in several works (Miyano et al. 1994; Case et al. 1998; Oh and Yoon 1995). Miyano and colleagues (1994) demonstrated that "the reciprocation law of time and temperature" is applicable for both the static and fatigue strengths, and that both the fatigue and static fracture modes and the slope of the S–N curves remains the same in the large temperature range. Oh and Yoon (1995) derived a formula for the fatigue life using the Zhurkov-type static life equation as well.

The purpose of this work is to investigate the effect of the loading history in cyclic loading on the damage evolution and lifetime of materials using the kinetic theory of strength (Hertzberg et al. 1975; Yokobori 1968, 1978; Mishnaevsky 1996, 1997; Hsiao 1989; Cherepanov 1974; Regel et al. 1974) and the stepwise representation of the loading variation during the fatigue cycles. As differed from the level crossing approach to the fatigue modeling suggested by Holm and de Mare (1988) and applied by Holm, de Mare and colleagues (Holm et al. 1995; Svensson 1996), we assume here that not the stress changes, but the acting constant stresses cause damage growth in materials. This assumption has been confirmed experimentally in many tests for static loading (Regel et al. 1974), and can be therefore used as a basis for the modeling of fatigue. Here only the case of relative low loading frequency ranges is considered, when the dynamic effects as well as the heat dissipation do not play any role.

2 Kinetic model of failure applied to the time-dependent loading

Let us consider a specimen under constant tensile stress (Fig. 1a). It has been shown in many works (e.g., Suresh (1998), Cherepanov (1974), Regel et al. (1974), Miyano et al. (1994), Case (1998), Oh and Yoon (1995), Holm and de Mare (1988), Holm et al. (1995), Svensson (1996), Narisawa et al. (1978)), that the lifetime of a specimen under constant load is an exponential function of applied stress and temperature:

$$t_F = A \exp\left(-B\frac{\sigma}{kT}\right) \quad (2)$$

where t_F, time-to-fracture; σ, applied stress; a and c, kinetic constants of material; k, Boltzmann constant; T, temperature.

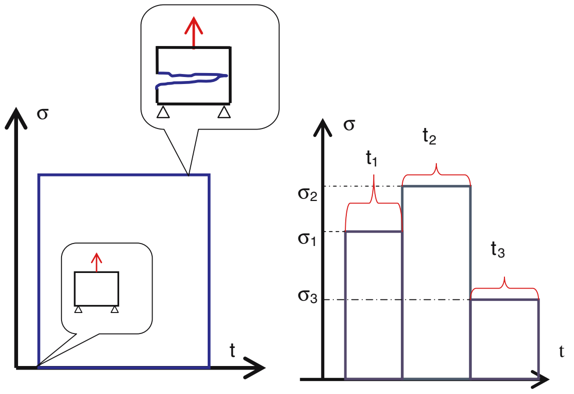

Fig. 1 Schemas of the constant (**a**) and multistep (**b**) loading of a specimen

This formula has been derived by several authors on the basis of the analysis of the accumulation of broken atomistic bonds whose breakage is caused by thermofluctuational processes (Palmgren 1924), or on the basis of the kinetic theory of failure (Yokobori 1968, 1978). Zhurkov introduced a kinetic concept of strength of solids, where time to rupture follows an Arrhenius–Eyring law with an energy barrier decreasing with increasing stress, and the driving mechanism for subcritical damaging processes is thermal activation (Palmgren 1924). Some versions of this formula were suggested by Hsiao (1989), and Cherepanov (1974). Oliveira (1998) further investigated and justified the kinetic model of fracture theoretically. Regel et al. (1974) carried out experimental investigations of this interrelation and determined material parameters for this curve.

Consider now a more complex case of multi-step loading, shown in Fig. 1b. Apparently, a failure of a material is not a step-wise event after a lapse of time, but continuous process of the defect accumulation and degradation at the lower scale level. In the case, shown in Fig. 1b, the failure does not occur after each loading step (since the duration of the steps is much lower than the time-to-failure for a given constant loading). However, such multi-step loading can lead to the failure as well as the one long step loading. Following Mishnaevsky and Schmauder (1997), Mishnaevsky (1998), let us define the damage degree in a material R as a function of the relation between the remaining and the total lifetime of an undamaged material

$$\mathrm{R} = \mathrm{t}/\mathrm{t_F} = 1 - \mathrm{t_{rem}}/\mathrm{t_F} = 1 - (\mathrm{t_{rem}}/A)\exp\left(B\frac{\sigma}{kT}\right) \tag{3}$$

where $\mathrm{t_F}$ is determined by the formula 2, t, current time (duration of loading); $\mathrm{t_{rem}}$, remaining time until failure. Thus, the total failure ($\mathrm{t} = \mathrm{t_F}$) takes place when the damage degree R reaches the critical value 1. When the load is first applied, the value R is equal to zero. The residual lifetime of a specimen under loading decreases due to the formation of defects.

Thus, in the case shown in Fig. 1b, the damage parameter increases as:

$$\mathrm{R} = \mathrm{R_1} + \mathrm{R_2} + \mathrm{R_3} = \frac{t_1}{t_F(\sigma_1)} + \frac{t_2}{t_F(\sigma_2)} + \frac{t_3}{t_F(\sigma_3)}, \tag{4}$$

where $\mathrm{t_F}(\sigma)$ is the function of the lifetime versus the applied stress, given by the formula 2. The formula 4 is in fact similar to the well-known Miner's rule.

Therefore, the residual lifetime of the material after the loading shown in Fig. 1b is (assuming that the specimen will be loaded by some constant load σ_4):

$$\mathrm{t_{rem}} = \mathrm{t_F}(\sigma_4)(1 - \mathrm{R}). \tag{5}$$

Using this model, we may study the effect of the loading history on the residual strength of materials. For the case of compressive loading, the parameters A and B in the formula 2 are different.

3 Fatigue of materials and frequency effect

Now let us consider fatigue of materials, given as cyclic tension-tension loading, shown in Fig. 2. By discretizing the loading curve, we can represent it as the multi-step loading, which is in principle similar to that shown in Fig. 1b. The model developed in Sect. 2 is applicable to this type of loading.

Consider the effect of the loading frequency on damage evolution in this case. Consider one half-cycle of the curve on Fig. 2. We represent the time-dependence of the loading in the form:

$$\sigma = \mathrm{at}, \tag{6}$$

where $\mathrm{a} = \mathrm{d}\sigma/\mathrm{dt}$ = rate of the loading growth at the half-step.

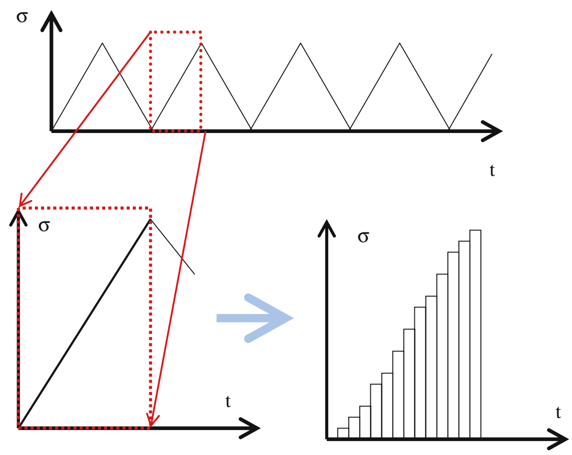

Fig. 2 Cyclic loading: representing of a half-cycle as a multistep loading

In this case, the stress amplitude is $\sigma_m = at_{cycle}$, where t_{cycle}, one half cycle duration, and the relation between the stress amplitude σ_m and the loading frequency f is given by formula:

$$f = 1/2t_{cycle} = a/2\sigma_m \tag{7}$$

Discretizing the half-cycle curve into M steps, we determine the damage increase for each ith step as:

$$R_i = t_i/t_F(\sigma_i) = (t_i/A)\exp(B\sigma_i/kT), \tag{8}$$

where $t_i = t_{cycle}/M$, and is independent on i, $\sigma_i = ait_i$.

The damage increase as a result of the half-cycle is:

$$R_c = \sum_i^M R_i = \sum_i^M (t_{cycle}/MA)\exp(aBit_{cycle}/kTM). \tag{9}$$

Taking into account the formula 7, we obtain the relationship between the damage growth in each cycle D_c and the frequency of loading:

$$R_c = \sum_i^M \frac{1}{2fMA}\exp\left(\frac{aiB}{2fMkT}\right) \tag{10}$$

One can see from this formula that the damage growth rate is a decreasing function of the loading frequency.

Transforming the summation from Eq. 10 into integration ($M \rightarrow \infty$, $t_i \rightarrow dt$), we have from Eq. 8:

$$\begin{aligned} R_c = \sum_i^M \quad R_i &= \sum_i^M (t_i/A)\exp(B\ at/kT) \\ &= \frac{1}{A}\int \exp\left(\frac{aBt}{kT}\right) \\ &= (kT/ABa)\exp(aBt_{cycle}/kT), \end{aligned} \tag{11}$$

and

$$R_c \propto \frac{1}{f}\exp(aB/2fkT). \tag{12}$$

If, according to the Miner's rule, we define the amount of cycles to failure as $N_F = 1/2R_c$, one may see from this formula that the number of cycles to failure increases with increasing the loading frequency.

For instance, if we take $A = 7.0 \times 10^6$ s (=200 h), $B = 5.5\,E^{-29}$ J/Pa, time to failure t_F (at the load 200 MPa) is about 100 h. For this case, we calculated the dependency of the number of cycles to

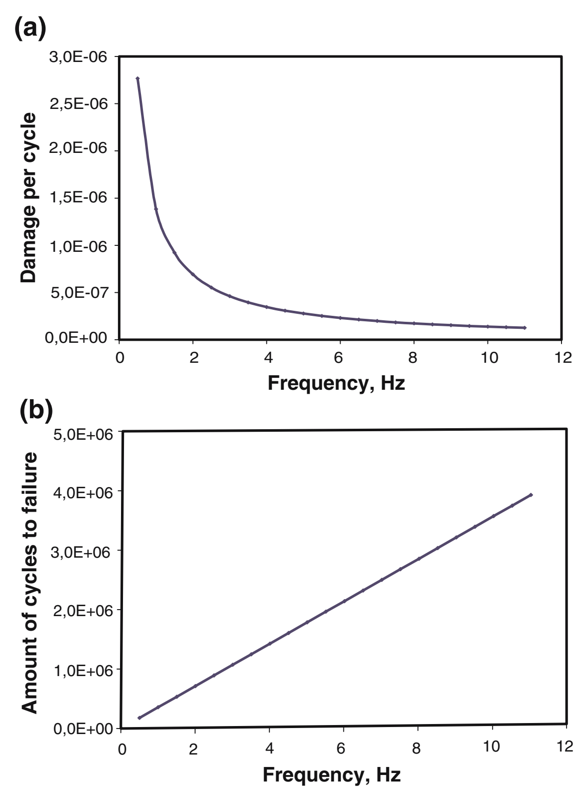

Fig. 3 The damage increase per cycle (**a**) and the amount of cycles to failure (**b**) plotted versus frequency of loading

failure on the frequency of cyclic loading. Figure 3 gives the curves of the damage increase per cycle and the amount of cycles to failure plotted versus frequency of loading ($\sigma_m = 300$ MPa, T = 270 K).

Using the formulas 10–12, we can determine the Wöhler (*S–N*) curve of the material. Defining the failure condition as $2R_cN = 1$, we obtain:

$$N = \frac{ABf\sigma_m}{kT}\exp\left(-\frac{B\sigma_m}{kT}\right) \tag{13}$$

This formula is obtained for the case of the triangular loading wave, shown in Fig. 2. For the case of the squared wave, the *S–N* formula takes the form:

$$N = 2fA\exp\left(-\frac{B\sigma_m}{kT}\right) \tag{14}$$

where it is assumed that the duration of each loading is 1/2f.

It is of interest that the total time to failure is constant: i.e., for the linear damage accumulation law, the frequency of loading does not affect the total time to failure.

For the sake of clarity, we would like to list and to summarize the assumptions, on which the model is based. The frequency of cyclic loading must be sufficiently low that both dynamic and thermal effects are negligible. The damage in the material is controlled by kinetic processes, and is therefore proportional to the duration of loading (weighted by the applied stress in each loading moment). In order to obtain closed-form expressions, we made several simplifying assumptions: the evolution of damage occurs linearly with time, and relatively low number of cycles. These assumptions are not inherent to the model and can be simply removed, if the model is generalized.

4 Stiffness reduction due to the microcracking and the lifetime of a material

In this paper, we used the definition of the damage degree in the material as a parameter of residual lifetime. One can note here that the damage parameter in the continuum damage mechanics is usually assigned two meanings: first, microcracks density, and second, the closeness of the material to failure. The conditions of failure are formulated usually as an equality between the damage parameter and some critical value (see for example, Lemaitre (1992)). Kachanov (1987) has formulated two meanings of damage parameter as follows: "reduction of the effective elastic stiffness" and "the extent of progression towards the final fracture." The latter meaning corresponds evidently to the parameter R (relative residual lifetime), defined in the Sect. 2.

Let us now establish relationships between the value R, defined in the Sect. 2, and the damage parameter D, defined via the microcrack density or the deterioration of elastic stiffness.

The damage growth law, derived by Lemaitre (1992), has a form:

$$\frac{dD}{dt} = \frac{C}{(1-D)^2}, \tag{15}$$

where C, a function of the stress state and material parameters; $C = (\sigma_{eq}^2 HR_v/2Es)\sqrt{(2/3)\dot{\varepsilon}_{p,\,i,\,j}\dot{\varepsilon}_{p,\,i,\,j}}$; s, energy strength of damage (material constant); H = 1, if the accumulated plastic strain reached the damage threshold; and H = 0, otherwise; ν, Poisson's ratio; R_v, triaxiality function; $R_v = (1+\nu)/3 + 3(1-2\nu)(\sigma_m/\sigma_{eq})^2$. The Lemaitre's damage parameter is defined as the relative reduction of the load bearing section of a specimen, and can be determined as

$$D = 1 - E_{dam}/E,$$

where E_{dam} and E, elasticity moduli of undamaged and damaged material. Integrating the Eq. 15, we derive the following cubic equation:

$$D^3 - 3D^2 + D - \int C dt = 0, \tag{16}$$

Taking the value C to be constant (over the loading period, compare Fig. 1a), we may substitute t instead of $\int C dt$ in this formula.

Solving this cubic equation, we obtain the damage parameter D as a function of loading time:

$$D = 1 - (1 - Ct)^{1/3} = 1 - (1 - CRt_F)^{1/3}, \tag{17}$$

The reduction of relative stiffness at each cycle is therefore:

$$dD/dN = D_c = 1 - (1 - C/2f)^{1/3} \tag{18}$$

Similarly to the results by Lee et al. (2003), the increase of damage D in each cycle is the more, the less the frequency of loading.

Assuming that if $D = 1$, $R = 1$, the formula 17 is reduced to

$$D = 1 - (1 - R)^{1/3}, \tag{19}$$

Using Eq. 17, we may relate the reduction of the material stiffness due to the microcracking, and the relative reduction of lifetime:

$$E_{dam} = E(1 - CRt_F)^{1/3}. \tag{20}$$

Apparently, the stiffness of the material decreases when the remaining lifetime decreases.

Figure 4 shows the value of the Lemaitre damage D (relative reduction of the material stiffness) plotted versus R. One can see that while the remaining lifetime is almost proportional to the stiffness of damaged material at the initial stages of the damage evolution, the microcrack density grows at the last stages of destruction with almost no effect on the remaining lifetime.

It is of interest to compare the curve on Fig. 4 with the results on the damage versus percent of life for fiber reinforced polymer composites (Reifsnider et al. 1990). The linear-plateau-linear D-R curve reflects three stages: the multiple fiber cracking (first linear part), crack coupling and delamination growth (plateau) and fracture (the second linear part). In our case, the intensive fracturing of the main bearing elements (fibers) at the initial stage of loading, leading to the quick weakening of the material, is not included into the model (the model is developed for a general case, not for a fiber reinforced composites). That is why the curve shown in Fig. 4 consists of two parts: slow, plateau-like material degradation (accumulation of defects, first without interaction, then with weak interaction), passing into the quick, autocatalytic growth of largest crack(s).

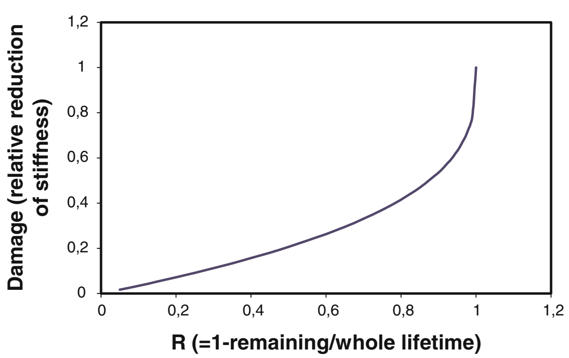

Fig. 4 Lemaitre damage D (relative reduction of stiffness) plotted versus R (1 – ratio of the remaining/total lifetime)

5 Composite materials

On this stage, let us estimate the effect of loading on the stiffness of the composites.

We consider a long fiber reinforced composite loaded along the fiber direction (see Fig. 5). In this case, the stress in the material is determined, according to the Voigt equation, by the formula:

$$\sigma_{comp} = v_f \sigma_f + \sigma_m v_m, \tag{21}$$

where v_f, v_m—volume content of fibers and matrix, respectively. Assuming that the fibers are linear elastic, and the matrix is viscoelastic, we have:

$$\sigma_{comp} = v_f E_f \varepsilon_{comp} + \sigma_m(\varepsilon_{comp}, E_m) v_m, \tag{22}$$

where $\sigma_m(\varepsilon_{comp}, E_m)$, stress as a function of the strain. If the matrix behavior can be modeled as a Kelvin-Voigt element, $\sigma_m = E_m \varepsilon_{comp} + \beta \dot{\varepsilon}_{comp}$, where β, damping coefficient.

Substituting (22) into (21), we have:

$$\sigma_{comp} = (E_{comp,0} - D_f E_{f0} v_f - D_m E_{m0} v_m)\varepsilon_{comp} + \beta \dot{\varepsilon}_{comp} v_m, \tag{23}$$

where $E_{comp,0}$, initial Young modulus of the non-damaged composite. One may see that if only fibers become damaged ($D_m = 0$), the stresses in the

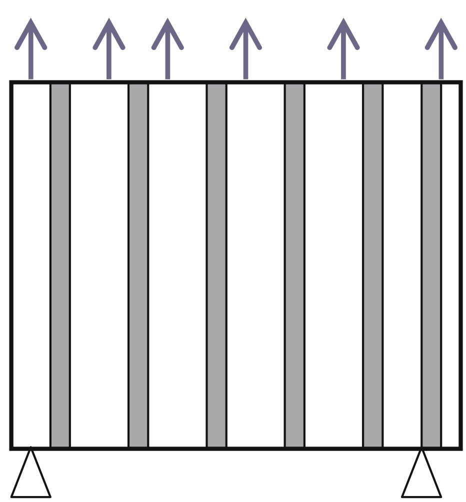

Fig. 5 Fiber reinforced composite: model loading

material are higher when the viscosity of the material increases.

Let us consider the effect of the availability of multiple constituents in the composite on its lifetime. The formula 1 was derived on the basis of both experimental data and probabilistic reasonings (accumulation of broken bonds in the material).

Now, we consider the failure of macroelements of the material (fibers and matrix between fibers) as random events as well. As "elements" in this case, fibers and the layers of matrix between the nearest fibers are taken. At this stage of work, only composite with strong interfaces are modeled, thus, only fibers and the section of matrix between fibers can fail.

According to the reliability theory, the reliability function (which is defined as a probability that an object does not fail in the time interval (0, t)) is calculated as:

$$\begin{aligned} \mathrm{Rel(t)} &= 1 - \mathrm{Prob_F} = \exp(-\mathrm{t}/\mathrm{t_F}) \\ &= \exp\left[-(t/A)\exp\left(B\frac{\sigma}{kT}\right)\right], \end{aligned} \tag{24}$$

where $\mathrm{Prob_F}$, probability that an object fails in the time interval (0, t) (Mishnaevsky 2007). One should note that the formula 24 is derived on the basis of the assumption about exponential probability law for the time-to-failure. While this assumption has some justification (e.g., (Mishnaevsky 1998)), other probability laws can be considered here as well.

Let us take a matrix between m fibers and the fibers around it, as a unit cell of material. In the case of the cell loaded along the fiber direction, it may be considered as a parallel system made from $m+1$ elements. For a simplest cell with m = 3 fibers and the matrix between them, shown in Fig. 6, the reliability function is given by formula:

$$\mathrm{Rel_{sys}} = 1 - (1 - \mathrm{Rel_f})^{\mathrm{m}}(1 - \mathrm{Rel_m}). \tag{25}$$

Assuming that all the elements of the system fail independently (which is not correct, of course, due to the load redistribution after a fiber or the matrix become damaged (Phoenix and Beyerlein 2000; Mishnaevsky 2007)), we can determine the mean time to failure of the parallel system under constant loading:

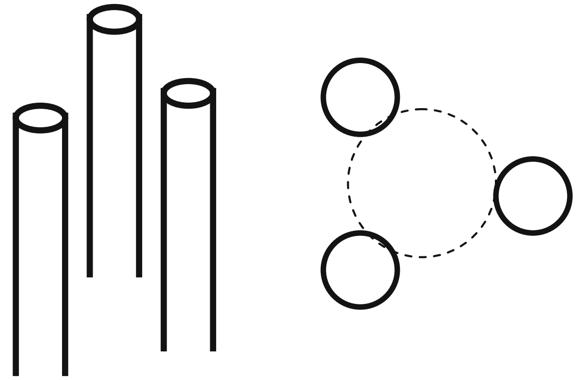

Fig. 6 Parallel system with three elements: matrix with three fibers around it (side view and section)

$$\mathrm{t_F} = \mathrm{t_{Fm}} + \beta \mathrm{t_{Ff}} - [1/\mathrm{t_{Fm}} + 1/(\beta t_{\mathrm{Ff}})]^{-1},$$

$$\text{where} \quad \beta = \sum_{\mathrm{i=1}}^{\mathrm{m}} (1/\mathrm{i})$$

Apparently, the relation with Eq. 25 is correct for the constant stress applied during the time t (the case shown in Fig. 1a). Consider now again the case shown in Fig. 1b. In this case, assuming the exponential reliability function, we have for a single fiber:

$$\begin{aligned} \mathrm{Rel(t_1+t_2+t_3)} &= \exp\left[-\frac{t_1}{t_F(\sigma_1)} - \frac{t_2}{t_F(\sigma_2)} - \frac{t_3}{t_F(\sigma_3)}\right] \\ &= \exp(-\mathrm{R_1} - \mathrm{R_2} - \mathrm{R_3}), \end{aligned} \tag{26}$$

For the case of the fatigue loading (considered in the Sect. 3), the probability that a single fiber does not fail during the half-cycle of fatigue loading, is given by the formula:

$$\begin{aligned} \mathrm{Rel_f(t_{cycle})} &= \exp(-\mathrm{R_c}) \\ &= \exp\left[-\sum_{i}^{M} (\mathrm{t_i}/\mathrm{A_f}) \exp(\mathrm{B_f}\sigma_{\mathrm{f,\,i}}/\mathrm{kT})\right]. \end{aligned} \tag{27}$$

where the index "f" means fiber. Substituting the index "m" for "f", we can obtain a similar formula for the matrix section between the fibers.

Substituting Eq. 27 into Eq. 25, and taking take into account the formulas 21–22 and the load sharing (following the effective stress concept (Lemaitre 1992)), we can calculate the reliability (probability of non-failure) of a given system.

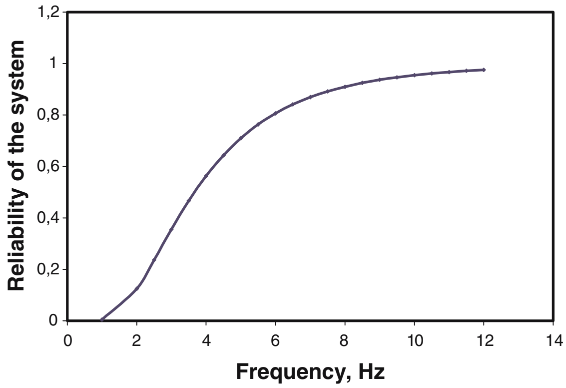

Fig. 7 Reliability of the system "4 fibers + matrix" after 10^7 loading cycles plotted versus the frequency of loading

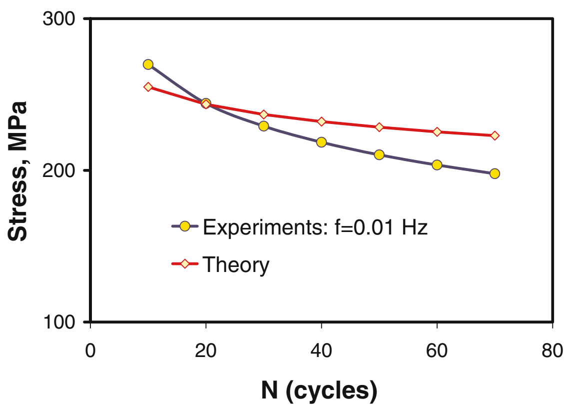

Fig. 8 Comparison of experiments and theory, for the case of frequency 0.01 Hz

To give an example of the effect of the loading frequency on the reliability of the m fibers-matrix system after many cycles of loading, let us consider the following case. The matrix material, considered above, is reinforced by fibers of another, stronger material (with the parameter $A_f = 2A_m$, where $A_m = A = 7.0 \times 10^6$ s is equal to the value given above; all other parameters the same). Taking $m = 4$, $\sigma_m = 300$ MPa, $T = 270$ K, we can obtain the reliability function after 10^7 loading cycles of loading as a function of the loading frequency. Figure 7 gives the curve for the considered model case.

6 Comparison with the experimental data

Let us compare the results of the kinetic model of fatigue damage with some experimental results.

Table 1 Regression formulas for *S–N* curves obtained in Mandell and Meier (1983)

Frequency (Hz)	Regression formulas for the *S–N*-Curves
1.0	$S = 405 - 45.0 \log N$
0.1	$S = 378 - 40.7 \log N$
0.01	$S = 355 - 37.0 \log N$

The model, developed above, is applicable to the case when neither dynamic effects nor heat dissipation influence the damage growth in the materials. Thus, we use the experiments by Mandell and Meier (1983) to verify this model. Mandell and Meier carried out the tension fatigue tests of plates constructed of eleven unidirectional plies of alternating 0 and 90°. The plates were subject to square and spike loadings. The tests "were run at low frequencies of 1.0, 0.1, and 0.01 Hz to prevent any hysteretic heating." Mandell and Meier (1983). As Mandell and Meier noted, "at higher frequencies there is an interaction of heating and mechanical effects, which . . . was intentionally avoided here." In Mandell and Meier (1983), both tensile cyclic loading and static experiments for cross-ply E-glass/epoxy laminates are presented. From the static experiments, one can determine the parameters A and B of formula 2. The linear regression formula for the static fatigue test was obtained in the form:

$$S = 369 - 16.5 \log t.$$

From this formula, we obtain the following values for A and B for the considered material:

$$A = 5.139 \times 10^9 \text{ s}; B/kT = 0.0606.$$

The *S–N* curves obtained by Mandell and Meier for different frequencies (0.01, 0.1, and 1 Hz), are presented in Table 1.

Figure 8 shows the *S–N* curve calculated on the basis of the developed model, and its comparison with the experimental data by Mandel and Meier. It should be noted that *S–N* curves are plotted with the independent variable (stress) on the *y*-axis. If we calculate the error on the basis of a fixed stress, not a fixed number of cycles to failure (what is the correct approach in this case), the error (overprediction) is up to 100% (i.e., for a stress of ∼250 MPa the data indicates N = ∼35 whereas the model claims N = ∼70). Practically, it means that while the developed approach allows the qualitatively correct prediction of the tendencies and *S*–*N* curves, a correction factor should be introduced to use this model for design purposes.

Using the formulas 13 and 14, one can estimate the effect of the wave shape (squared vs. triangular) on the N-S curve:

$$\frac{N_{squar}}{N_{tria}} = \frac{2kT}{B\sigma_{\rm m}} \tag{28}$$

Mandell and Meier (1983) considered the squared and triangular (spike) shapes of loading weaves, and obtained the following regression formulas for the *S*–*N* curves (f = 0.1 Hz):

$$\text{Square wave : S} = 446 - 49.9 \log \text{N} \tag{29}$$

$$\text{Spike loading: S} = 508 - 58.8 \log \text{N} \tag{30}$$

The ratio between the values of N for squared and triangular loading is 1.09 ... 1.12. Using the formula 28, we can calculate this ratio. Substituting all the values, we have:

$$N_{squar}/N_{tria} = 1.14 \ldots 1.16. \tag{31}$$

Thus, the estimation on the basis of formula 28 gives the results which are very close to the experimental results.

7 Conclusions

On the basis of the kinetic theory of strength, a new approach to the modeling of material degradation in cyclic loading has been suggested. Assuming that not stress changes (as in Holm and de Mare (1988); Holm et al. (1995); Svensson (1996)), but acting stresses cause the damage growth in materials under cyclic loading, we applied the kinetic theory of strength to model the material degradation in fatigue. The damage growth per cycle, the effects of the loading frequency on the lifetime and on the stiffness reduction in composites were determined analytically. It has been shown that the number of cycles to failure increases almost linearly and the damage growth per cycle decreases with increasing the loading frequency.

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