Project funded by the European Commission under the 6th (EC) RTD Framework Programme (2002- 2006) within the framework of the specific research and technological development programme "Integrating and strengthening the European Research Area"

Project UpWind.TTC

Contract No.:
019945 (SES6)

"Integrated Wind Turbine Design"

MODEL OF FATIGUE MICRO DAMAGE IN FR COMPOSITE

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APPROVER:	

Document Information

DOCUMENT TYPE	Deliverable
DOCUMENT NAME:	Model of fatigue micro damage in FR composite
REVISION:	
REV.DATE:	
CLASSIFICATION:	R1: Restricted to project members
STATUS:	

Abstract: The continuum model of interface damage onset and accumulation in FRC has been suggested based on the established correlation between the fiber arrangement and the peak interface stress statistics. The method combines the multipole expansion technique with the representative unit cell model of random structure FRC able to simulate equally well the uniform and clustered random fiber arrangements. By averaging over a number of numerical tests, the empirical probability functions have been obtained for the nearest neighbor distance and the peak interface stress. It is shown that the considered statistical parameters are rather sensitive to the fiber arrangement, in particular, cluster formation. An explicit correspondence between them has been established and an analytical formula linking the micro structure and peak stress statistics in FRC has been suggested. Application of the statistics of extremes to the local stress concentration study has been discussed. It is shown that the peak interface stress distribution in FRC with uniform micro structure follows Fréchet-type asymptotic distribution rule. The presented numerical data demonstrate potential of the developed approach: practical importance of the established relationships (as well as other analogous dependencies which can be obtained in this way) consists in the following.

The ultimate goal of our simulation consists in development of the continuum theory of FRC strength. To accomplish this task, we need to link the micro structure parameters to the peak local stress statistics and micro damage initiation and accumulation rate. The statistical parameters of an actual FRC micro structure and the constants entering the local stress distribution functions we found from the numerical experiments would be the input variables of this theory.

Contents

STATUS, CONFIDENTIALITY AND ACCESSIBILITY									
Status				**Confidentiality**				**Accessibility**	
S0	Approved/Released			**R0**	General public			Private web site	
S1	Reviewed			**R1**	Restricted to project members	**R1**		Public web site	
S2	Pending for review			**R2**	Restricted to European. Commission			Paper copy	
S3	Draft for commends			**R3**	Restricted to WP members + PL				
S4	Under preparation			**R4**	Restricted to Task members +WPL+PL				

PL: *Project leader* **WPL:** *Work package leader* **TL:** *Task leader*

1. Introduction

Theoretical prediction of fiber reinforced composite (FRC) strength is challenging because the micro damage onset is governed by the peak local stress rather than by its mean value. Except the very dilute case, interaction between the fibers results in that stress concentration on them deviates from that observed for a single fiber embedded in an unbounded matrix and, due to randomness of the structure of composite, is a random function of spatial coordinates. The peak stress location and level is rather sensitive to the arrangement of fibers so their reliable prediction requires an adequate account for a micro structure statistics of actual FRC and interactions between the fibers. The problem becomes much more tough in the case of composite with statistically non-uniform fiber distribution.

The papers by Pyrz (1994a,b) and Pyrz and Bochenek (1998) address the problems of quantitative description of arrangements and correlations in unidirectional FRCs. It has been indicated there that both topological and second-order statistics are related to the local stress variability under transverse loading conditions. However, their stress analysis should be regarded as qualitative because the iterative model they applied assumes constant stress inside the fiber. It allows mean stress evaluation in a weakly heterogeneous material; however, peak local stress even in two-fiber problem cannot be captured this way. As shown by Ganguly and Poole (2004), the iterative model can only be applied to dilute composites where the minimum inter-fiber distance exceeds the fiber radius three and more times.

A promising way to account for the fiber arrangement statistics and interactions between them is known in the mechanics of composites as the "representative unit cell" approach. It is based on modeling the structure of an actual heterogeneous solid by a periodic medium whose representative unit cell contains a number of inhomogeneities. This approach is advantageous in that the quasi-random micro structure of such a material, with a prescribed statistical structure parameters, can be specified explicitly. Moreover, due to deterministic nature of this model, it is possible to formulate and solve the periodic model problem accurately and thereby to account for the interactions among the inclusions in a rigorous manner.

In the framework of UPWIND.TTC Project, an efficient multipole expansion technique-based method and numerical code have been developed (Kushch et al., 2008) for evaluating the micro stress field in a fibrous composite. The method combines the superposition principle, theory of complex potentials and Fourier series expansion in order to reduce the meso cell model problem to an ordinary, well posed set of linear algebraic equations. By averaging over a number of random structure realizations, the statistically meaningful results have been obtained for both the local stress and effective elastic moduli of disordered fibrous composite. In the present study, the method by Kushch et al. (2008) is applied to develop the micro damage model of FRC based on the relationship between the fiber arrangement type and the local peak interface stress statistics in FRC. To this end,

- the representative unit cell model of fibrous composite able to simulate both the uniform and clustered random fiber arrangements is used;
- a series of computational experiments has been carried out and the empirical probability functions have been obtained for the nearest neighbor distance and the peak interface stress. It is shown that the considered statistical parameters are rather sensitive to the fiber arrangement, in particular, cluster formation;
- an explicit correspondence between them has been established and the relevant analytical formulas have been written. It is found that the peak interface stress distribution in FRC with uniform random arrangement of fibers follows Fréchet rule;
- based on the above findings, the micro damage model of FRC has been suggested.

2. Representative meso cell model of fiber reinforced composite (FRC)

2.1. *Model geometry*

Reliable prediction of FRC micro damage onset and accumulation requires (a) using a geometry model statistically close to the micro structure of an actual FRC and (b) an adequate account for interactions between the fibers. It, in turn, necessitates statistical description of both the geometry and local stress field because in most FRCs, both micro structure and stress field are the random functions of spatial coordinates. On the other hand, the local damage level and strength limit of FRC is governed by the peak local stress, location of which is quite sensitive to the fiber arrangement. The probability theory formalism seems to be an adequate and probably the only consistent way for linking the statistical parameters of micro structure and peak local stress distribution.

To meet above requirements, we use the many-fiber meso cell model of the FRC bulk, shown in Fig. 2.1. Specifically, we consider a meso cell model of FRC (Kushch et al, 2008) containing a number of aligned and circular in cross-section fibers. The whole composite bulk is obtained by translating the cell in two orthogonal directions. Number N of the fibers with centers inside the cell is taken large sufficiently to approach micro structure of an actual disordered composite.

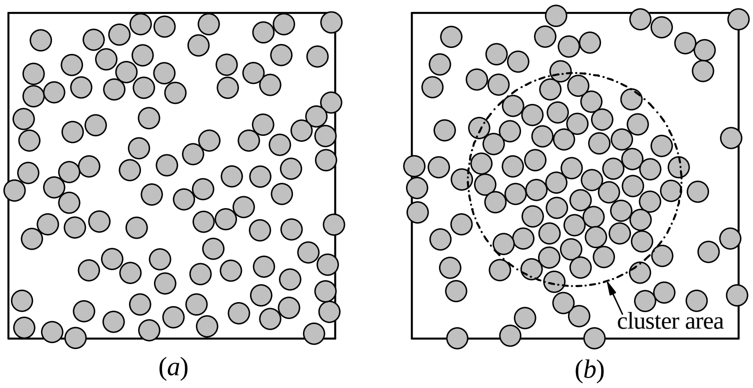


Fig. 2.1. Meso cell model of the FRC bulk with (a) uniform and (b) clustered micro structure

The model provides simulation of composites with non-uniform distribution of fibers as well. Specifically, we consider the cluster of fibers of circular shape shown in Fig. 2.1b: its size is defined by the radius or, alternatively, by the volume content of clusters. Due to presence in this model of an additional structure parameter, it can be regarded as a meso level model; for more discussion on the subject, see Mishnaevsky (2007).

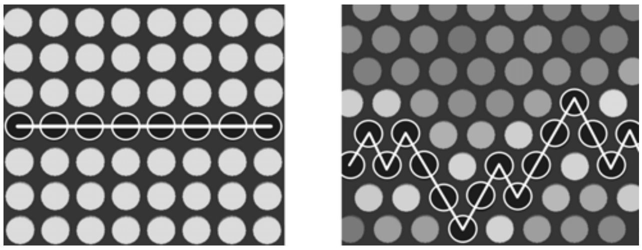
2.2. *Model boundary value problem (BVP) and analytical method*

Within the 2D framework, the plane strain, plane stress and anti-plane shear problems for FRC model are studied. Both the matrix and fiber materials are isotropic and linearly elastic. At the matrix-fiber interfaces, the perfect bonding conditions (continuity of displacements and normal tractions) are prescribed. The stress field in the composite bulk is assumed to be macroscopically homogeneous, which means constancy of the volume-averaged, or macroscopic, strain and stress tensors, the last one considered as the governing parameter. Under macroscopic stress homogeneity condition, periodicity of structure results in periodicity of relevant physical fields. The stress periodicity is regarded as the cell boundary condition providing continuity of the displacement and stress fields between the adjacent cells. Decomposition of displacement field involves the linear part being the far field and the periodic fluctuation caused by the inhomogeneities.

An accurate analytical method developed by Kushch et al (2008) to solve for stress in the meso cell comprises the Muskhelishvili (1953) complex variables technique, the superposition principle, the Golovchan et al. (1993) singular periodic potentials, the Fourier and Laurent series and the newly derived re-expansion formulae. The solution of the model BVP is built in a computationally cost-efficient way. Fulfilling all the boundary conditions reduces the model problem to the well-posed infinite set of linear algebraic equations with the matrix coefficients given by rational expressions and involving (unlike FEM or BEM) no integration. It provides high numerical efficiency of the method. A detailed account of the method is given in the *UpWind ISM Annual Report-2007*.

2.3. *Numerical study: debonding path model*

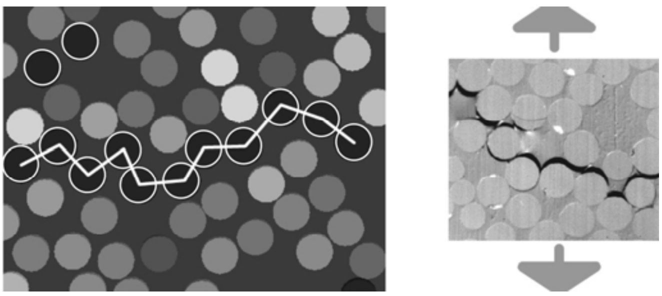
Now, we simulate progressive damage in FRC using the meso cell model assuming the matrix-fiber interface to be the "weakest link". The constant amplitude cyclic load applied to the composite part results in the continuous dropping down the interface strength. The debonding occurs when the peak interface stress $\sigma_m = \max_{\varphi} \sigma_{rr}$ reaches the strength limit on the most heavily loaded fiber. An instantaneous and complete fiber debonding is assumed: the debonded fiber is replaced with a pore and the modified model BVP is solved again, etc. In Fig. 2.2, the typical debonding fiber paths in the regular structure FRCs are shown obtained by the step-by-step progressive damage algorithm. In the perfectly bonded composite of regular structure, the stress field is identical for each fiber, so we artificially introduce the local defect (a single debonded fiber). It causes the stress re-distribution between the adjacent fibers and the next and all subsequent debonding fibers are determined uniquely. The debonded fibers form a chain transverse to the loading direction: for a square array (Fig. 2.2a), this chain is a straight line row whereas in the hexagonal structure a zigzag-like chain, with more debonded fibers involved, is developing.



(a) (b)

Fig. 2.2. Debonding path in the regular structure FRC: (a) - square, (b) - hexagonal packing

A similar debonding pattern is observed in the random structure FRC. In Fig. 2.3a the typical simulation result is given whereas Fig. 2.3b shows the experimentally observed (Gamstedt and Andersen, 2001) crack path. The analogous experimental data have been obtained by other authors (e.g.,Tsai, 1988). For more details, see Kushch et al (2008).



(a) (b)

Fig. 2.3. Debonding path in the random structure FRC: (a) - simulation,
(b) - experiment by Gamstedt and Andersen (2001)

As seen, our model predicts correctly the damage pattern. In order to get a quantitative estimate of damage in terms of applied loads, number of cycles, stiffness reduction, etc., an additional effort in development of the model is required including interface and matrix cracking, delamination and other observable fatigue damage events. Another issue is randomness of structure which necessitates statistical post-processing of this kind simulation.

3. Peak interface stress in FRC

3.1. *Peak stress statistics in the random structure FRC*

Now, we proceed to statistical analysis of local stress field in FRC. Specifically, we study an effect caused by fiber arrangement on the peak interface stress σ_m. Assuming this stress responsible for local damage onset, one can think of the "interface strength affected by micro structure". To obtain the test-invariant statistical distributions, σ_m were averaged over 50 runs at a given fiber volume content *c*. The empirical probability function $F(\sigma)$ obtained by computer simulations for c from 0.1 to 0.5 is shown in Fig. 3.

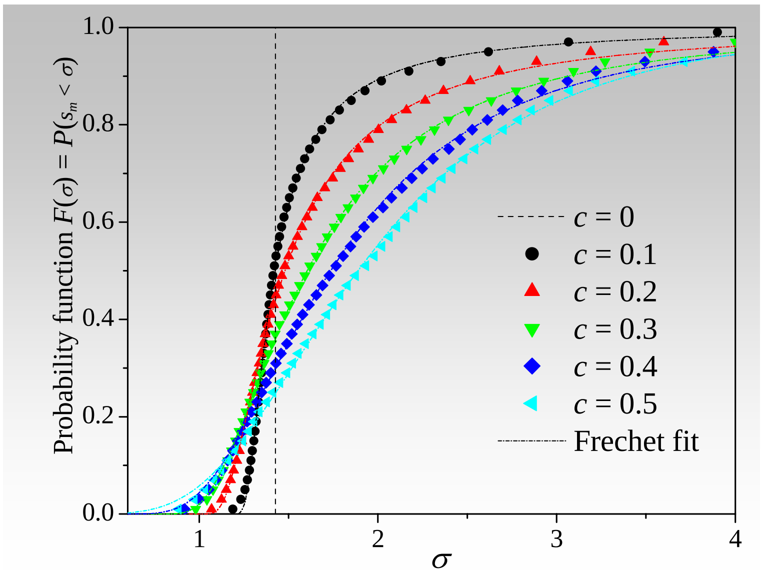


Fig. 3.1. Empiric probability function of the peak interface stress σ_m: an effect of fiber volume content

Numerical study shows that there always (regardless of c) exists a relatively low fraction of fibers with high interface stress. The maximum stress is localized between the closely placed fiber pairs and exceeds greatly the mean stress value. In terms of interface strength it means that debonding will occur in these "hot spots" much earlier than in the other sites. This observation correlates well with - and can be quite plausible explanation of - the experimentally observed (Brøndsted et al., 1997; Talreja, 2000; Van Paepegem and Degrieck, 2002; among others) rapid FRC stiffness degradation due to matrix-fiber debonding at the initial stage of cyclic loading.

3.2. *Peak interface stress and statistics of extremes*

Then, an intriguing question arises: does so close correlation between the peak stress statistics obtained from numerical tests and the known statistical distributions is simply a matter of luck? - or, possibly, this is manifestation of a certain intrinsic rule. To answer this question, we refer to the statistical theory of extreme values (called briefly as "statistics of extremes") being modern and rather promising branch of the probability theory (see, e.g., Beirlant et al., 2004). One of the principal results of this theory is "Three Types Theorem" (Gnedenko, 1943). It asserts that if a distribution function does not put all its mass at a single point, it must be one of three types:

Gumbel-type: $\Pr[X \le x] = \exp\{-\exp[-(x-\mu)/\sigma]\}$, all x; (3.1)

$$\text{Fréchet-type: } \Pr[X \le x] = \begin{cases} 0, & x < \mu; \\ \exp\left[-\left(\dfrac{x-\mu}{\sigma}\right)^{-\xi}\right] & x \ge \mu; \end{cases} \tag{3.2}$$

$$\text{Weibull-type: } \Pr[X \le x] = \begin{cases} \exp\left[-\left(\dfrac{x-\mu}{\sigma}\right)^{\xi}\right], & x \le \mu; \\ 1, & x > \mu. \end{cases} \tag{3.3}$$

This theorem gives a rigorous theoretical substantiation to the known already empirically found statistical distributions. What is important, it also hints what kind of distribution is expected in each specific case.

It is quite clear that our problem can be thought in context of the statistics of extremes. With account for that the far field load is uniaxial tension, the conservative bound for σ_m is $\sigma_m > 0$. It makes Fréchet distribution (3.2) a plausible analytical form of the stress cumulative probability function. The corresponding best fit curves are shown in Fig. 3.2 by the dashed lines. Indeed, Fréchet fit practically coincides with the results of computer simulation. At the same time, an attempt to use Gumbel distribution (3.1) as a fitting function (dash-dotted curve) cannot be considered as successful. It is seen from the plot that discrepancy is quite significant and its maximum is observed for high stress, responsible for strength and, therefore, the most interesting for us area. As to "short-tailed" Weibull-type distribution (3.3), it fails completely to approximate the data obtained from numerical experiment.

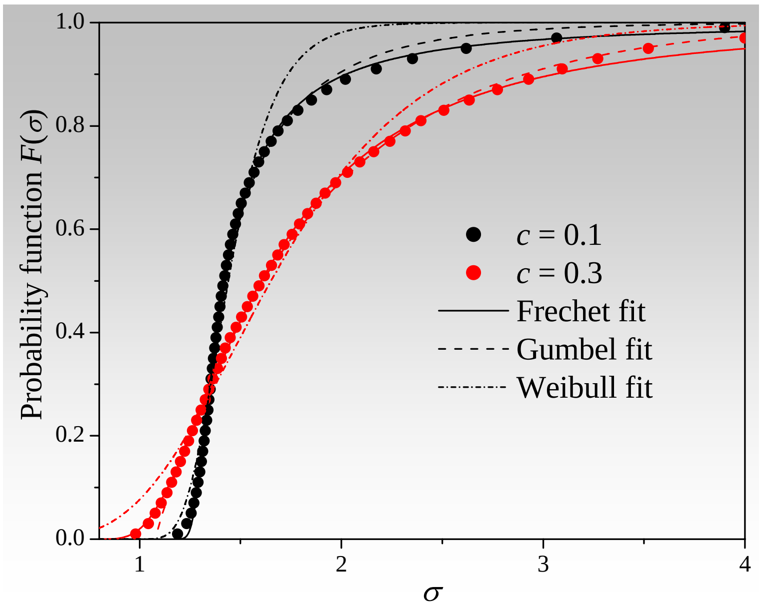


Fig. 3.2. Fréchet, Weibull and Gumbel distributions as the fitting functions

The conclusion is made that with a high degree of probability, the peak interface stress distribution in FRC with uniform random arrangement of fibers follows Fréchet rule. Using the simulation data, the Fréchet distribution parameters have been found as the functions of fiber volume content *c*:

$$F_s(\sigma) = \exp\left\{-\left[p_3(c)/(\sigma - p_1(c))\right]^{p_2(c)}\right\}. \tag{3.4}$$

4. Micro structure - peak stress correlation

4.1. *Micro structure statistics: nearest neighbor distance*

Stress field in and around a given fiber is greatly amplified by the surrounding fibers, and the peak stress location and magnitude are sensitive to the fiber arrangement type. Reliable prediction of σ_m distribution requires an adequate account for the micro structure statistics of actual FRC. The fiber arrangement can be characterized by several parameters, including coordination number, particle cage, inter-particle spacing, second-order intensity (Ripley's) function, radial distribution function, nearest neighbor distribution function, two-point cluster function, etc. (Pyrz, 1994b; Babuška et al., 1999; Torquato, 2002; Buryachenko et al, 2003; among others). At the same time, a very little is known about relationship between the statistics of structure and local fields. Probably, the most definite (qualitative, however) conclusion made there consists in that σ_m amplification and its variability is strongly affected by the nearest neighbor distance d_m and nearest neighbor orientations of adjacent fibres and therefore the stress amplification is more pronounced for the clustered structure as compared with the statistically uniform one. An important and challenging question arises regarding the a_m - d_m correlation in a FRC with random micro structure.

To answer this question, we use the nearest neighbor statistics for packing of hard disks (Torquato, 1995). There, for the exclusion probability function $E(r)$ equal to the probability that a circular region of radius r encompassing the reference fiber is free of other fiber centers, the following analytical representation was found:

$$E(x,c) = \exp\left\{-c\left[4a_0(c)\left(x^2-1\right)+8a_1(c)(x-1)\right]\right\} \qquad (4.1)$$

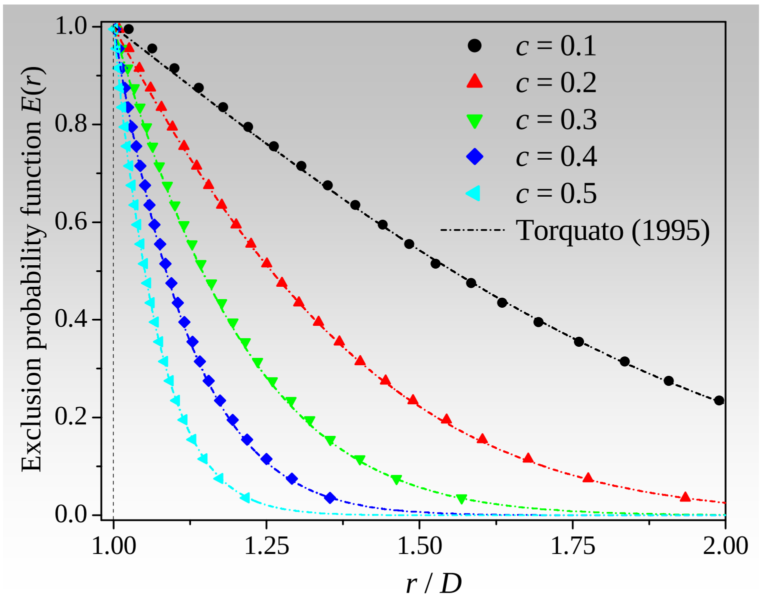


Fig. 4.1. Exclusion probability function: numerical simulation and approximation

Comparison of the results calculated according to (4.1) with our numerical results on the nearest neighbor statistics is illustrated in Fig. 4.1. There, the symbols represent the empirical exclusion probability function obtained by averaging over 50 realizations of random structure for a given fiber volume content c. The results of computer simulation agree closely with the theory which validates the algorithm and computer code of cell geometry generation. On the other side, it proves (4.1) to be an analytical description of statistically uniform random micro structure FRC.

4.2. *Stress concentration vs nearest neighbor distance*

Both σ_m and d_m are the random numbers: no direct functional dependence is expected between them. At the same time, such a relationship can be easily found between the

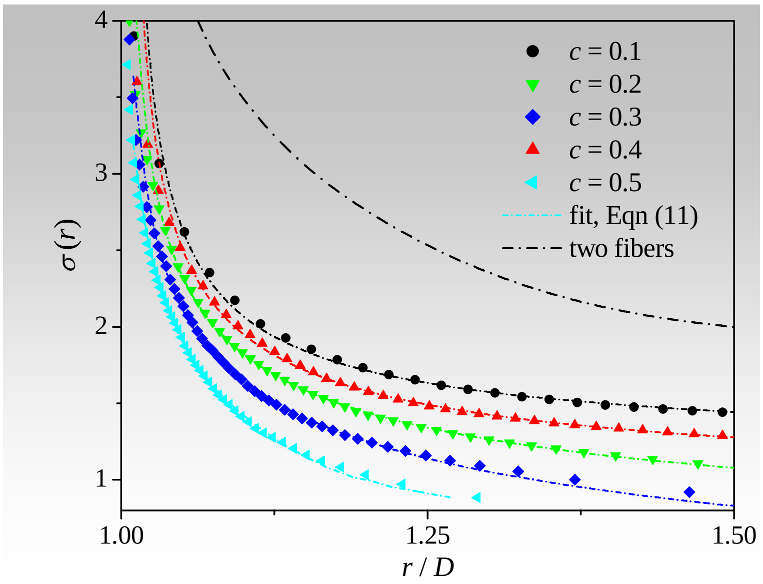


Fig. 4.2. Peak interface stress vs inter-fiber distance: effect of fiber volume content

arguments of the relevant probability functions, $F(\sigma)$ and $E(r)$. Namely, we equate the probability of finding the neighbor fiber at the distance larger than d_m to the probability of that the peak interface stress σ_m does not exceed a certain value σ. Combining the formulas (3.4) and (4.1) gives an explicit formula

$$\sigma_m(x,c) = p_1(c) + p_3(c)\left\{c\left[4a_0(c)\left(x^2-1\right)+8a_1(c)(x-1)\right]\right\}^{-1/p_2(c)} \qquad (4.2)$$

The empirical dependencies $\sigma(r)$ obtained by matching the numerical data in Fig. 3.1 and Fig. 4.1 are shown in Fig. 4.2 by the discrete symbols; the dash-dotted lines represent the eqn (4.2).

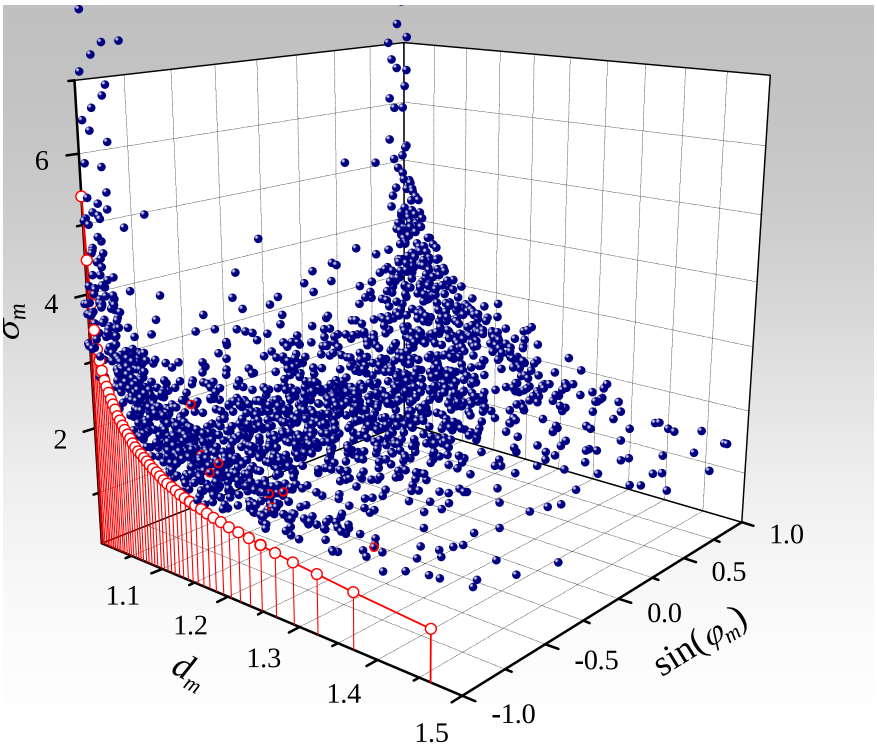


Fig. 4.3. Peak interface stress vs nearest neighbor distance and orientation

As seen from the plot, agreement between the numerical data and their analytical representation is close. The observed in Fig. 4.2 tendency of decreasing the peak stress with *c* growing up is anticipatory: the higher is the fiber volume content, the less room left for the isolated clusters of a few fibers where the high interface stress concentration is most probable.

The formula (4.2) deserves a certain criticism because d_m is the leading, but in no way the only factor affecting σ_m. Another important structure parameter is the nearest neighbor orientation (e.g., Pyrz and Bochenek, 1998) characterized by the angle φ_m between the line connecting the centers of nearest fibers and loading direction. Correlation between σ_m and φ_m is seen from Fig. 4.3, where the raw data of computer simulation for c = 0.4 are shown. An angular dependence of peak stress is clearly seen: as expected, the highest σ_m is observed in the fiber pairs with φ_m close to $\pm \pi/2$. For comparison, the corresponding curve $\sigma(r)$ taken from Fig. 4.2 is shown in Fig. 4.3 by the open circles. The conclusion has been made that equation (4.2) gives a reasonable σ_m approximation for φ_m close to $\pm \pi/2$ where the highest stress and hence local damage is expected.

4.3. *Effect of clustering*

Now, we consider local stress fields in FRC containing the fiber clusters of circular shape and estimate an effect of the cluster volume fraction c_{cl}. To be specific, we put volume content of fibers in the cell and cluster c = 0.3 and c_{in} = 0.5, respectively: $c = c_{in}\, c_{cl} + c_{out}\,(1- c_{cl})$, where c_{out} is the fiber volume content outside the cluster: for the clusters with c_{cl} = 0.3 and c_{cl} = 0.5, we get c_{out} = 0.21 and c_{out} = 0.10, respectively.

An effect of clusters on the exclusion probability function is seen from Fig. 4.4, where the empirical function $E(r)$ obtained by computer simulation is shown by the solid and open circles.

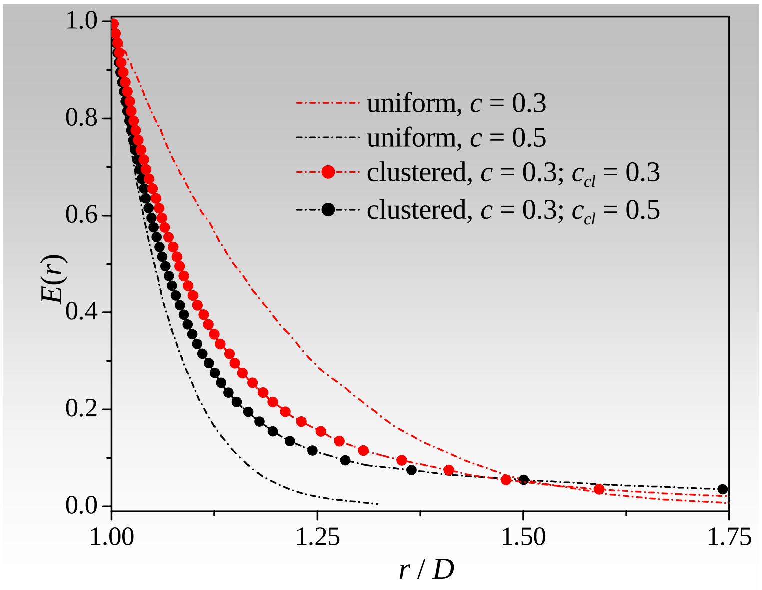


Fig. 4.4. Exclusion probability function vs inter-fiber distance: effect of cluster volume content

Here, we plot also $E(r)$ for FRC with uniform micro structure and c = 0.3 (dashed line) and c = 0.5 (dash-dotted line). As comparison shows, it is rather sensitive to the non-uniformity in spatial distribution of fibers. Expectably, a clustered structure contains a larger number of closely placed fibers. As a result, the initial slope of $E(r)$ is close to that of uniform structure with c = 0.5 rather than c = 0.3 (remind, that the overall fiber volume content in clustered FRC is 0.3).

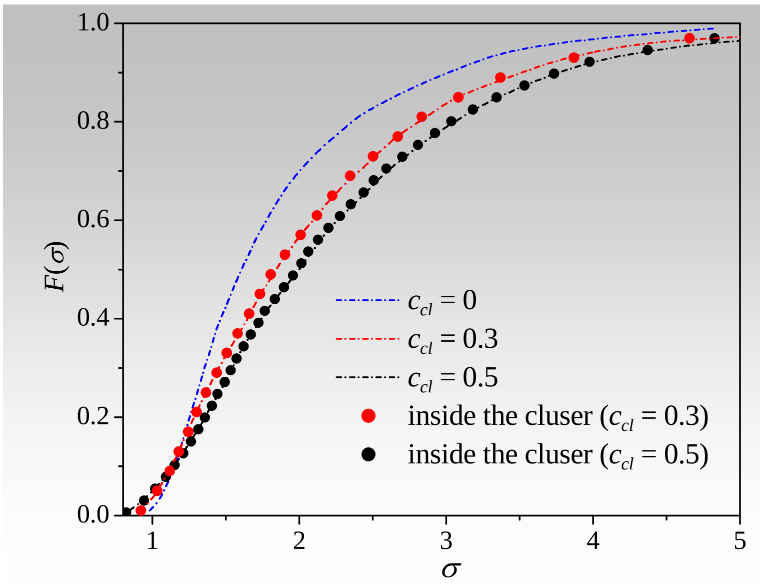


Fig. 4.5. Peak interface stress distribution: an effect of the cluster volume content

In Fig. 4.5, the empirical probability function $F(\sigma)$ (3.4) is shown calculated for the uniform and two clustered arrangements of fibers. The dash-dotted (c_{cl} =0.3) and solid (c_{cl} =0.5) curves deviate significantly from the dashed line representing the statistically uniform random structure. The data in Fig. 4.5 confirm the observation made by Pyrz (1994a) that the mean value of max radial stress shifts markedly towards higher values for clustered patterns. It is noteworthy, however, that the empirical function $F(\sigma)$ built over the cluster area and shown by the solid and open triangles in Fig. 4.5 is statistically indistinguishable from that one built over the whole cell area and shown by dash-dotted and solid lines. It means that clusterization increases the peak interface stress on the fibers inside and outside the cluster simultaneously.

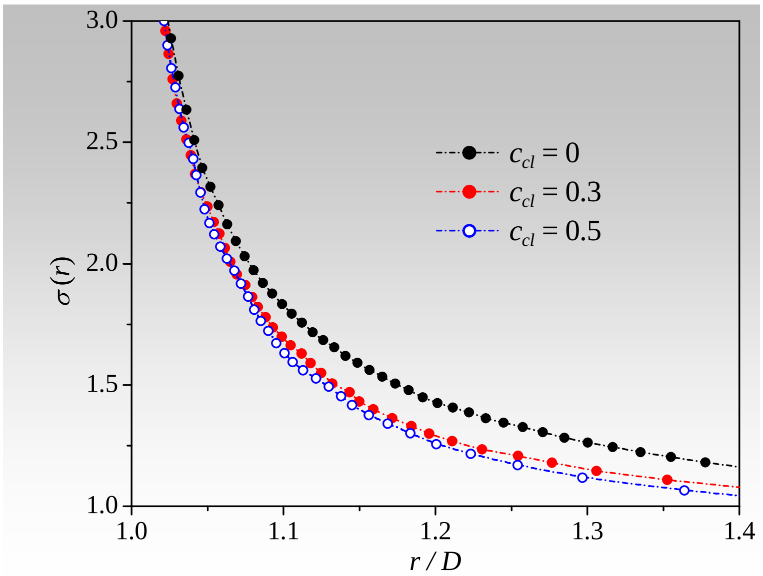


Fig. 4.6. Peak interface stress vs inter-fiber distance: an effect of clusterization

By analogy with Fig. 4.2, we match the numerical data in Fig. 4.4 and Fig. 4.5 to get the $\sigma(r)$ for a clustered structure, see Fig. 4.6. The difference between the uniform and clustered structure is clearly observable. The "universal" $\sigma(r)$ curve, valid for any fiber arrangement type is improbable: instead, we expect it depending on the meso parameters of non-uniform structure.

5. Micro damage model of FRC

Practical significance of the developed approach to the peak stress statistics study consists in that it provides the theoretical framework for the micromechanical theories of FRC strength. Below, we consider one theory of this kind to show the way of incorporating available statistical information in the continuum damage model. We assume (a) matrix-fiber interface the "weakest link" in FRC and (b) debonding the only micro damage type. Damage criterion is taken in the form $s_m = \max_{\varphi} \sigma_{rr} = \sigma^*$ where the interface strength $\sigma^* = const$ for brittle fracture and $\sigma^* = \sigma^*(N_c) \sim N_c^{-1/m}$ for fatigue, N_c being a number of loading cycles. Alternatively, one can consider σ^* varying randomly from fiber to fiber, with a known statistical rule.

An elementary damage event is an interface crack onset so it seems natural to consider the interface crack density $D = N_{db} / N_{tot} = \Pr\left[s_m \geq \sigma^*\right]$ as a damage parameter. Provided N_{tot} taken sufficiently large,

$$D = 1 - \Pr\left[s_m < \sigma^*\right] = 1 - F_s\left(\sigma^* / P\right) \tag{5.1}$$

This formula estimates the damage level in terms of applied load and interface strength σ^*. Assuming interface degradation due to cyclic loading, one can write the damage accumulation rule in FRC as

$$D(N_c) = 1 - \exp\{-[p_3 / (N_c^{-1/m} / \langle\sigma_{22}\rangle - p_1)]^{p_2}\} \tag{5.2}$$

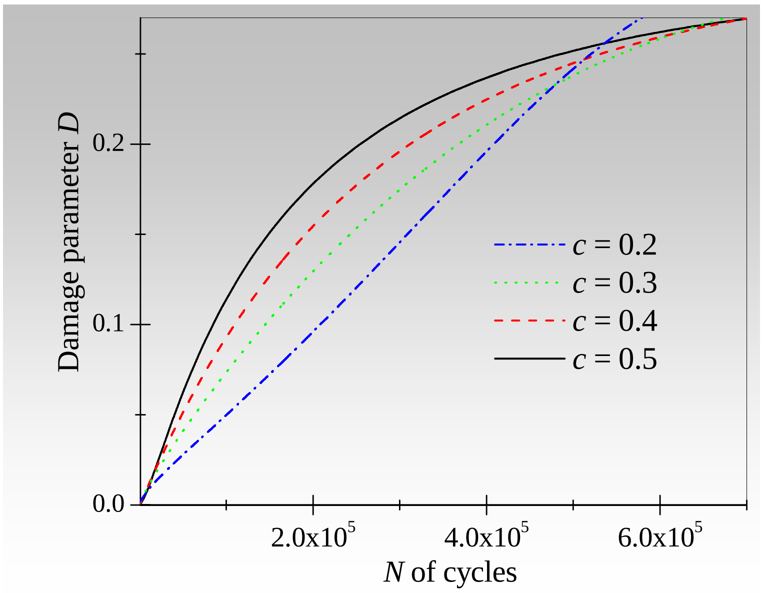


Fig. 5.1. Interface damage growth of FRC due to cyclic loading

In (5.1) and (5.2) we imply that D is sufficiently low and interactions between the cracks can be neglected. I.e., this model describes an early stage of interface damage development, accompanied by rapid stiffness reduction (e.g., Bronsted et al., 1997; Van Paepegem and Degrieck, 2002). At low D, an effect of interface cracks on the effective elastic modulus of FRC can be approximated by

$$E^*(D) / E^*(0) \approx 1 - \beta D = \beta \exp\{-[p_3 / (N_c^{-1/m} / \langle\sigma_{22}\rangle - p_1)]^{p_2}\} - \beta \tag{5.3}$$

where β is a factor to be found experimentally or from simulation (Meraghni et al., 1996; Zhao and Weng, 1997; among others). Together with (5.2), (5.3) gives an estimate of stiffness reduction degree vs number of loading cycles. Predicted by our theory normalized effective

Young modulus for the fiber volume content c = 0.2, 0.3, 0.4 and 0.5 is shown in Fig. 5.2, where we put β=1.85 and m=3. As seen from the plot, theory reproduces, at least, qualitatively, the experimental observations by Bronsted et al. (1997) and Van Paepegem and Degrieck (2002). For more details, see Kushch et al (2009).

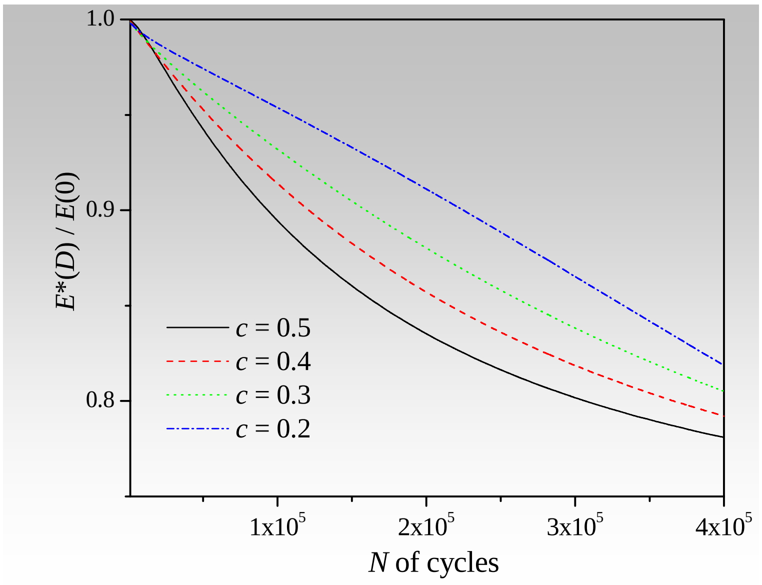


Fig. 5.2. Stiffness reduction of FRC due to cyclic loading

For each specific composite material, there is a number of factors affecting its strength but not taken into account in our theory. The considered model can be generalized in many ways, including (a) loading type, (b) interface debonding criterion, (c) fatigue law, (d) residual (setting) stress, etc. Incorporation of these (as well as other analogous) features makes the model more realistic: at the same time, it necessitates conducting a separate series of numerical experiments. The developed approach does not introduce any simplifying assumptions regarding the stress fields. In contrast to available continuum theories of FRC micro damage, we deal with the local, rather than phase-averaged, stress which justifies application of the well-established strength criteria of phase materials and interfaces. Also, the proposed model provides a comprehensive account of microstructure and interactions between the fibers, captures the essential physical nature of the fatigue process and thus provides a reliable theoretical framework for a deeper insight into the fatigue damage initiation and accumulation phenomena in a fiber reinforced composite.

6. Conclusions

The continuum model of interface fatigue damage onset and accumulation in FRC has been suggested based on the established correlation between the fiber arrangement and the peak interface stress statistics. The method combines the multipole expansion technique with the representative unit cell model of random structure FRC able to simulate equally well the uniform and clustered random fiber arrangements. By averaging over a number of numerical tests, the empirical probability functions have been obtained for the nearest neighbor distance and the peak interface stress. It is shown that the considered statistical parameters are rather sensitive to the fiber arrangement, in particular, cluster formation. An explicit correspondence between them has been established and an analytical formula linking the micro structure and peak stress statistics in FRC has been found. Application of the statistics of extremes to the peak local stress study has been discussed. It is shown that the peak interface stress in FRC with uniform micro structure follows Fréchet-type asymptotic distribution rule.

Practical importance of the established relationships consists in the following. The ultimate goal of our simulations consists in development of the continuum theory of FRC strength. To accomplish this task, we need to link the micro structure parameters to the peak local stress statistics and micro damage initiation and accumulation rate. The statistical parameters of an actual FRC micro structure and the constants entering the local stress distribution functions we found from the numerical experiments would be the input variables of this theory.

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