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DEVELOPMENT AND TESTING OF COMPRESSIVE DAMAGE MODEL OF POLYMER FRP

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Contents

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Status				**Confidentiality**				**Accessibility**	
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Statistical modelling of compression and fatigue damage of unidirectional fiber reinforced composites

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ABSTRACT

A statistical computational model of strength and damage of unidirectional carbon fiber reinforced composites under compressive and cyclic compressive loading is presented in this paper. The model is developed on the basis of the Budiansky–Fleck fiber kinking condition, continuum damage mechanics concept and the Monte-Carlo method. The effects of fiber misalignment variability, fiber clustering, load sharing rules on the damage in composite are studied numerically. It is demonstrated that the clustering of fibers has a negative effect of the damage resistance of a composite. Further, the static compressive loading model is generalized for the case of cyclic compressive loading, with and without microdegradation of the matrix, and with and without random variations of loading. It was observed that the random variations of loading shorten the lifetime of the composite: the larger the variability of applied load, the shorter the lifetime.

1. Introduction

Compressive strength of unidirectional fiber reinforced composites is of great importance for many practical applications. The micromechanisms of compressive strength are different from the mechanisms of tensile or shear strength, and are strongly influenced by the microstructure imperfections, like fiber misalignment, waviness, etc.

The purpose of this work is to investigate the effect of microstructure and the statistical distribution of microstructural parameters of the composites on the damage, compressive and fatigue strength of unidirectional fiber reinforced composites. A statistical computational model of a composite with a number of randomly distributed and randomly misaligned fibers was developed. Testing this model (under compressive loading along the averaged fiber axis with different fiber arrangements and phase properties, under cyclic compression with constant, random increasing and decreasing loadings), we could analyze the effect of the microstructural parameters of the composites, like the fiber clustering, variability of fiber misalignment, matrix hardening, etc. on the compressive strength of the composite.

2. Modelling of compressive failure of composites: a brief overview

The damage mechanisms of unidirectional composites under compressive loading differ strongly from those under tensile loading [1–3], and require therefore the application of different modeling approaches. In many cases, kinking of fibers is the dominant compression failure mechanism [4,5]. According to Moran et al. [5], kinking of fibers can be separated into three stages: *incipient kinking* (microbuckling of fibers, caused by imperfections of microstructures and matrix shears), *transient kinking* (kink band propagation, unstable rotation of fibers within the band tip, and strong shear deformation of matrix, up to the locking the fibers in their orientation) and *steady-state kink band broadening.*

Traditionally, the *first stage* of the kinking, incipient kinking, has been modeled with the use of the analytical methods of *theories of elasticity* [6–9], and, later, *plasticity* [10–11]. Rosen [7,8] and Schuerch [9] pointed out to the role of elastic instabilities in the fiber buckling, and derived formulas for composite failure stress using the elastic microbuckling analysis. Argon [10] and Budiansky [11] included the effects of matrix plasticity and the initial misalignment of fibers into the analytical models, and determined the critical compressive stress as shear yield stress divided by the initial fiber misalignment angle. Their analyses were further generalized by including the cases of elastic plastic matrix [11] and the plastic strain hardening of the matrix [12]. Following the works by Argon and Budiansky, several authors considered the effect of imperfections of fiber shapes and orientation on the composite strength, e.g., the effects of sinusoidal regular and irregular fiber waviness [13], as well as sinusoidal imperfections with variable (decaying) amplitude [14], and combination of global (sinusoidal waviness of fibers along their axis) and local (sine wave added to a strip of many fibers at free left side) imperfections [15].

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Different methods have been used to derive formulas for the critical condition of kinking and buckling of fibers: on the basis of the classical beam theory [13], Rice's theory of the localization of plastic deformation [16–17], on the basis of bifurcation analysis and rate constitutive equations for fiber composites [16], Timoshenko shear deformation beam model [18], 3D finite element simulations of cylindrical fiber, embedded to cylindrical matrix [19].

In a series of works, the interaction and *competition between several damage mechanisms* (fiber splitting vs. kinking, matrix cracking vs. kinking) was considered. So, Jensen developed a non-dimensional criterion D, characterizing the failure mode (matrix cracking vs. fiber kinking) in the composite, by combining a unit cell model with the model of kinking based on the Rice's plastic localization theory [16,17].

A number of models of the later stages of kinking, the *propagation and broadening of kink bands* are based on the fracture mechanics methods [21–25].

The effects of local imperfections of microstructures seems to be one of the most important questions in predicting the compressive strength of composites: over decades, a growing attention is attracted to every kind of imperfection, and each considered imperfection (misalignment, waviness, etc.) plays an important role for the composite strength.

In many works, the fiber misalignment is assumed to be a constant value over the specimen. However, there are experimental evidences that the fiber misalignment is a random value and follows the Gaussian distribution law [26–28]. The effect of random distribution of fiber misalignments was taken into account by Barbero and his colleagues [26,27] in their models of compressive strength of composites. The analytical model of Barbero is based on the continuum damage mechanics approach, and the pre-defined approximate formula for the shear stress strain response of the matrix. However, the analytical model developed by Barbero can not be used to investigate the effect of different fiber arrangements, material laws of components and other microstructural effects on the compressive strength of composites. In order to solve this problem and to carry out parametric studies of the effect of microstructures on the compressive strength of composites, a computational model of composite with random fiber arrangement and random fiber misalignment has been developed. The model and the computational experiments based on this model are described below.

3. Statistical modelling of compressive damage

3.1. Multifiber model with random misalignments

In order to analyze the statistical effects in fiber kinking in composites, a computational code for the computational analysis of compression damage, based on the statistical model of fiber composite, damage mechanics and fiber kinking condition by Budiansky and Fleck, has been developed. The carbon fiber reinforced epoxy matrix was considered. The fibers are assumed to be elastic, while the mechanical behavior of the epoxy matrix is described by Ramberg-Osgood equation. It is assumed that the fiber misalignment is a random value with Gaussian distribution.

For the analysis, a unit cell with a pre-defined amount of fibers is generated in an interactive session. The fibers are randomly arranged in the cell, using the RSA (random sequential absorption) algorithm [2]. The misalignment angles are assigned to each fiber, using random normal number generator (with Gaussian probability distribution). The material properties used in the simulations are shown in Table 1. Then, the unit cells were subject to axial loading (or repeated loadings). For each fiber, the kinking condition is checked, according to the Budiansky–Fleck kinking condition [12].

If one or several fibers kink, the stress is redistributed over remaining fibers, thus, increasing the load on remaining fibers, and the likelihood of their kinking.

3.2. Load redistribution after the fiber kinking

In order to model the load redistribution after the kinking of one or few fibers, the following model was used. The damage parameter is calculated as the amount of kinked fibers divided by the total amount of fibers

$$D = M_{fail}/M. \tag{1}$$

The kinked fibers are assumed to carry no more load [26], or (a generalized version) to have reduced stiffness.

The averaged stress on the remaining intact fibers (after the kinking of one or few fibers) is calculated on the basis of the "effective stress concept" of damage mechanics [2]:

$$\sigma = \sigma_0/(1-D), \tag{2}$$

where σ_0 is the stress on a fiber before the kinking of one or more fibers. Further, the stresses on the fibers are redistributed depending on the distance between a kinked fiber and the considered intact fiber, according to the power load sharing law [29,30]:

$$\sigma \propto r^{-k}, \tag{3}$$

where σ – stress on a given fiber, r – distance between a failed and the considered fiber, k – power coefficient. Here, $k = 0$ corresponds to global, and $k \sim \infty$ to local load sharing rules [2].

In order to determine the stress concentration factor for intact fibers after failure of one or several fibers, we used the following reasoning. Presenting the formula (3) in the form

$$\sigma = ar^{-k},$$

where a – proportionality coefficient, and determining the total load on all fibers before and after fiber failure, we can obtain:

$$a = \frac{\sigma_0 M}{\sum_M r^{-k}}, \tag{4}$$

where M – amount of fibers, r_{ij} – distance between fibers i and j. Thus, the stress on i-th fiber after kinking of j-th fiber can be determined as

$$\sigma_i = \frac{\sigma_{av} M}{(1-D)\sum_M r^{-k}} r_{ij}^{-k}, \tag{5}$$

Here σ_{av} – averaged stress on the fibers (before the fiber kinking), $\sigma_{av}/(1-D)$ – averaged stress on the fibers (after the fiber kinking). Formula (5) was introduced in our program in order to take into account the load redistribution on fibers.

Table 1
Material properties used in the simulations (for sample material, AS4/E7K8).

Composite				Matrix				Fiber
Volume content of fibers (%)	Initial in plane/composite shear stiffness G_{lt}, GPa [26]	Composite shear strength τ_u, MPa [26]	Standard deviation of fiber misalignments, deg. [31]	Young modulus, GPa [26,34]	Poisson ratio [26,34]	Power coefficient in Ramberg–Osgood relationship [33]	T_y, in Ramberg–Osgood relationship, MPa [33]	Young modulus, GPa [32]
50	5354	157.5	1.15	3.79	0.37	3	35	276

Further, two scenarios of the load redistribution after the fiber kinking were considered, depending on the "rate" of the load redistribution: "quick" loading (when the fibers are loaded, and fail independently, and the load redistribution takes place only at the next loading), and "slow" loading (after a fiber is failed, the stress on the remaining fibers increases instantly according to the "effective stress concept" and "load sharing rule", and so on for all the fibers which fail successively one after another). In the first case ("quick" loading), a *j*th fiber does not "know" that the *i*th fiber failed, until the next cycle of loading. Only in the next cycle, the load is redistributed over remaining fibers. In the latter case, the fibers kink one after another, depending on misalignment of each fiber. Thus, the "slow" loading leads to autocatalytic fiber kinking, caused not by the increase of applied load, but rather by the load redistribution after the fiber begin to kink.

4. Computational experiments: static loading

In this section, we seek to analyze the effect of microstructural parameters of the composite (variability of misalignments, fiber clustering, matrix properties) on the damage evolution under static compressive loading.

4.1. Load sharing effect: "quick" versus "slow" loading of composite

Let us consider the effect of load sharing rule, and the "velocity" of the load redistribution on the damage evolution in composites. Following [29,30], we assume that the load sharing rule can be approximated by the power law (3).

In order to analyze the effect of power coefficient in the Eq. (3) on the damage growth in the composite, a series of damage simulations for the unit cells with 500 fibers and different values of power coefficients k were carried out. Fig. 1 shows the damage plotted versus applied stress for the different values of power coefficient in the loading sharing law, for the "slow" loading case. It can be seen that the formation of kind bands or kinking of sufficient fraction of fibers cannot be achieved even at very high loads in the case, if the load from failed fiber is distributed only to nearest neighbors ($k > 5$).

For the case of "quick" loading, the power coefficient of the load sharing equation does not influence the damage evolution at the first cycle of loading. However, it does influence the damage at the repeated loadings.

Fig. 2 shows the damage plotted versus the power coefficient of the load sharing equation, for the 1st and 2nd loading.

From this analysis one can see that the load sharing rule has a strong influence on the damage evolution in composites. The global load sharing leads to quicker damage growth. Thus, if we can localize the damage evolution (for instance, by placing weak fibers inside a group of strong fibers), we can delay the destruction of composite.

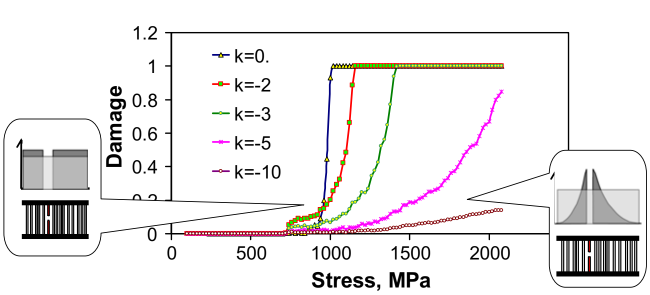

Fig. 1. Damage plotted versus applied stress for the different values of power in the loading sharing law ("slow" loading).

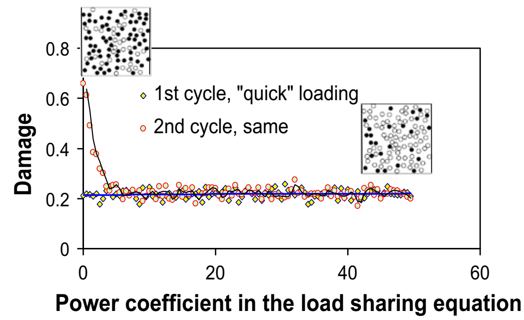

Fig. 2. Damage plotted versus the power coefficient in the loading sharing law ("quick" loading): 1st and 2nd loading cycle.

As noted by Zhou and Wagner [35], the local effect of fiber breaks on nearest neighbors decreases when matrix/fiber interface begins to debond, and when this debonding grows. Thus, the availability of interface debonding leads to the more global load distribution after fiber kinking, and, therefore, to the quicker damage growth.

For the subsequent simulations, we need to estimate the value of the power coefficient for the case of undamaged interface. According to the experiments carried out at the Risø National Laboratory for Sustainable Energy, the compressive strength of carbon/epoxy composite (65% carbon) is of the order of 1350 MPa [36]. Making the correction on the volume content of fibers (65%/50%), we obtain the estimation of the compressive strength in this case as 1040 MPa. Similar results have been obtained in [31] (1000–1200 MPa). Comparing this value with Fig. 1, one can see that this critical stress corresponds to the value $k = -2$. It is of interest that the value $k = -2$ was referred to in [30] as well. This value of k will be used in our further simulations.

In order to compare the damage growth curves for the "quick" and "slow" loadings, we carried out simulations for two cases. Fig. 3 shows the damage in the composite, calculated by formula (1) and plotted versus the stress on a fiber for the cases of "quick" and "slow" loadings. The volume content of fibers was taken 50%, and there were 500 fibers in the model. The power coefficient k in the Eq. (3) was taken to be -2.

4.2. Effect of the variability of fiber misalignment on the damage in the composite

In order to analyze the effect of the variability of the fiber misalignments on the damage in the composite, we carried out a series of simulations of compressive damage for unit cells with different variabilities of fiber misalignments. The standard deviation of the normal distribution of fiber misalignments was varied from 0.5° to 2.0°. The calculations were made for the case of 500 fibers,

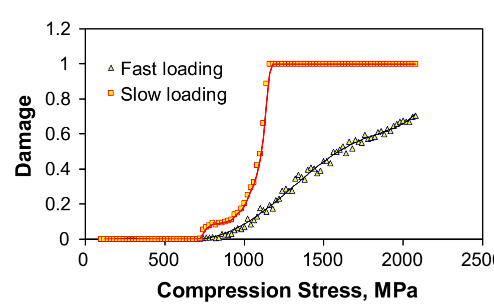

Fig. 3. Damage in the composite versus the stress on a fiber: "quick" and "slow" loading, $vc = 50\%$, $M = 500$, $k = 2$.

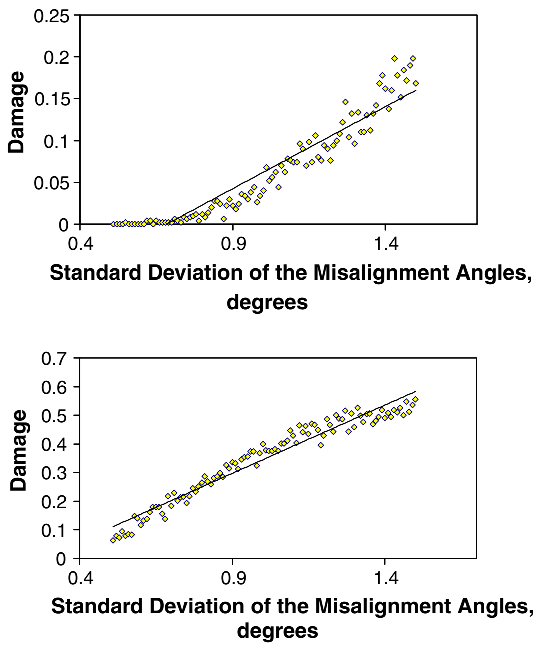

Fig. 4. Damage in the composite plotted versus the standard deviation of the normal distribution of fiber misalignments: (a) the stress on fiber 1000 MPa and (b) 1500 MPa.

two levels of applied stresses (1000 MPa and 1500 MPa), and the volume content of carbon fibers 50%. The loading was applied in the "quick" regime.

Fig. 4 shows the damage in the composite plotted versus the standard deviation of the normal distribution of fiber misalignments. As expected, the higher fiber misalignment leads to the quicker damage and stiffness loss of the composite.

4.3. Effect of fiber clustering on the kinking

The conclusion drawn above about the positive effect of the localized load sharing on the damage evolution could lead us to the assumption that the clustered arrangement of fiber (or so called fiber bundles) should have a positive effect on the composite strength. In order to verify this assumption, we carried out corresponding numerical experiments.

The clustered fiber arrangement was generated automatically, following the algorithm described in [2,37,38]. The number of clusters was pre-defined. Then, the centers of clusters were arranged in such a way that the distance between them is equal or more than the double cell size divided by the amount of clusters. The fiber centers were arranged at random, normally distributed distances from the cluster centers.

The models with random and clustered arrangements of fibers were subject to loadings ("quick" loading scheme). In order to make the clustering effect better visible, we considered the composite with 20% volume content of fibers.

In the simulations, it was observed that the fiber clustering has no effect on the damage at the first "quick" loading. However, at the second "quick" loading, the composites with clustered fiber arrangements demonstrated sufficiently higher damage.

Figs. 5 and 6 show the distributions of failed fibers in the cases of clustered and random homogeneous fiber arrangement, for the $N = 100$ (5 clusters) and $N = 500$ (10 clusters), after the second loading cycle. The applied load was 1200 MPa.

Fig. 7 shows the damage plotted versus applied stress, for the clustered and random homogeneous fiber arrangements, with 500 fibers, for 1st and 2nd loading cycles.

On the basis of the analysis, one can conclude that the clustered fiber arrangement leads to the quicker failure of composite, due to the effect of the load redistribution.

While there might be no difference between the clustered and homogeneous fiber arrangement if the material is no pre-damaged and is loaded quickly, the clustered arrangement leads to the much quicker failure of fibers at the second loading (or if the material is pre-damaged). For instance, at the compressive stress 1500 MPa, the damage in composite with clustered fibers is 32.5% higher than in the composite with homogeneously arranged fibers.

If fibers are clustered, failure of one fiber leads to the quick failure of all fibers in the cluster, as soon as the load redistribution effect takes place.

Let us compare these results with other results on the effect of fiber clustering for similar cases [2,39,40]. Sørensen and Talreja

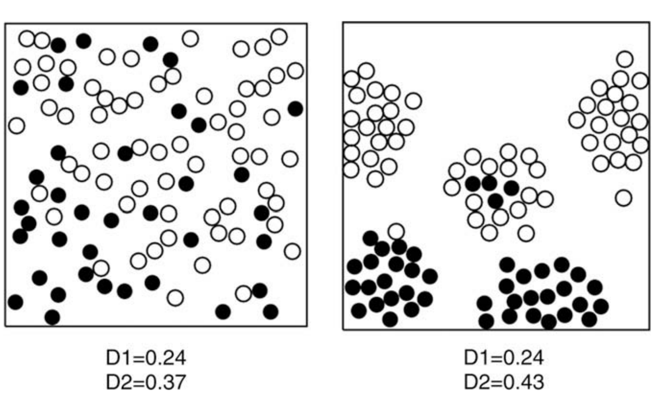

Fig. 5. Distribution of failed fibers in the case of clustered and random homogeneous fiber arrangement. The case of 100 fibers, 5 clusters, $vc = 20\%$.

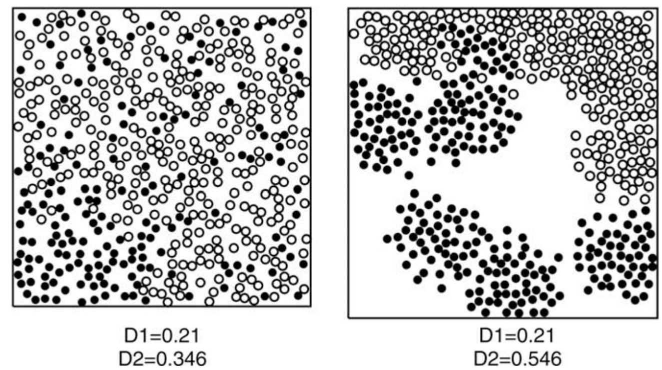

Fig. 6. Distribution of failed fibers in the case of clustered and random homogeneous fiber arrangement. The case of 1000 fibers, 10 clusters, vc = 20%.

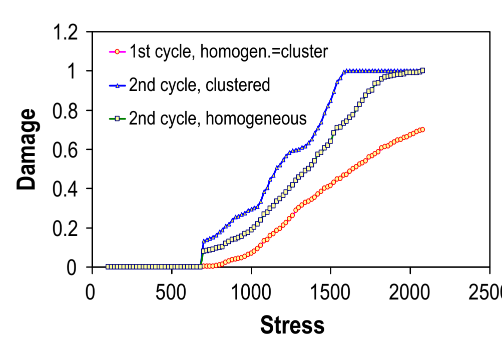

Fig. 7. Damage plotted versus applied stress, for the clustered and random homogeneous fiber arrangements.

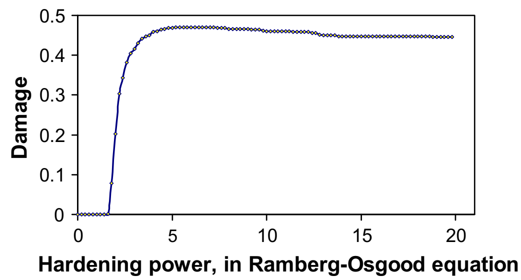

Fig. 8. Damage plotted versus the power coefficient of the Ramberg–Osgood law for the matrix hardening.

[39] observed that the fiber clustering leads to the higher compressive stresses in fibers. Segurado et al. [40] observed the increase in the reinforcement damage with increasing the reinforcement clustering in the composite.

4.4. Effect of the hardening behaviour of the matrix

Here, the effect of the hardening behavior of matrix on the damage in the composite is considered. The power coefficient n in the Ramberg–Osgood formula for the matrix hardening behavior was varied from 0 to 11. The results of damage in the composite are plotted versus the power coefficient n in Fig. 8.

From the Figure can be seen that the fiber kinking behaviour is influenced by the hardening law of the matrix: fiber kinking takes place only if $n > 1$. The density of kinked fibers is highest at $n = 2–3$ and is reduced up to 12% when n increases further.

4.5. Formation of the kink band

A cluster of adjacent, buckled/kinked fibers can be considered as a kink band. Fig. 9 shows the scheme of the formation of kink band as a percolation cluster from bucked fibers (top view). In order to determine whether the buckled fibers form an infinite, spanning cluster in a given section of the cell, the methods of the continuum percolation theory (Swiss cheese model) were used. The arrays of the coordinates of fibers, marked as kinked or intact, are given to the percolation subroutine. The fibers are considered as "adjacent" and forming a cluster, if the distance between their axes is no more than 3 fiber radii. The linear size of the kink band is determined as the distance between two most distant kinked fibers in the fiber cluster.

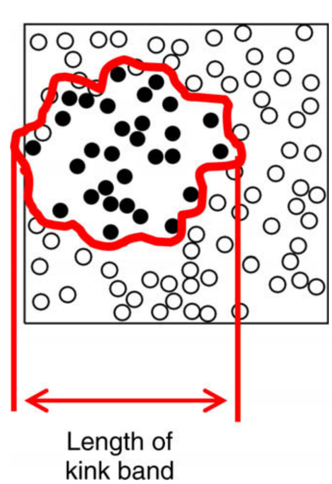

Fig. 9. Scheme (top view): formation of kink band as an percolation cluster from bucked fibers.

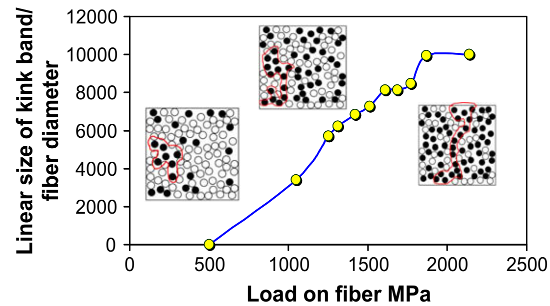

Fig. 10. Length of the kink band plotted versus the applied stress.

Fig. 10 shows the length of the kink band plotted versus the applied stress. The length of king band is normalized over the fiber diameter, which is taken 10 μm here.

The kink band begins to form at the load 500 MPa, and grows almost linearly in the stress range between 500 and 1500 MPa. After 1700 MPa, some slow down of the band growth is observed.

4.6. Touching fibers: critical volume content of fibers

Using the developed program code, we sought to analyze the effect of the volume content of fibers on the amount of touching fibers. Touching fibers are a quite common defect in composites, and are very dangerous for the fatigue strength of the materials. In the model, developed in Section 3, we assumed that the fibers can not contact, and set the minimum distance between two adjacent fibers to be $1.005r$ (r – radius of fiber). In this subsection, we replace this condition by the condition that the minimum distance between the axes of adjacent fibers is no less than 1.98 of the fiber diameter (thus, assuming that the 0.01 of fiber radius is taken by its surface roughness and interphases). Using this assumption, we calculated how many fibers touch one another at a given volume content of fibers. The condition that two adjacent fibers touch one another is formulated as a condition that the distance between axes is between 1.98 and 2.02 of the fiber diameter.

Fig. 11 shows the amount of touching fibers divided by the total amount of fibers plotted as a function of the fiber volume content.

One can see that, at the volume content approaching 60%, the density of touching fibers begins to increase quickly (the curve begins to grow almost vertically). Thus, the volume content of the order of 60% might be a critical level, at which the positive effect of strong reinforcement is balanced and eventually negated by the effect of the touching fiber defects. This conclusion is confirmed by experimental observations which showed that the fiber volume content of a composite 56–57% is a critical value, after which the composite might have rather high strength and stiffness, but very low fatigue resistance.

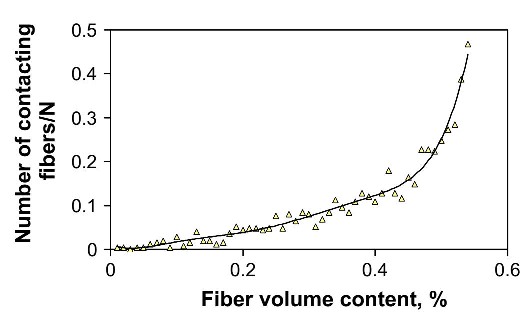

Fig. 11. Amount of touching fibers divided by the total amount of fibers plotted as a function of the fiber volume content.

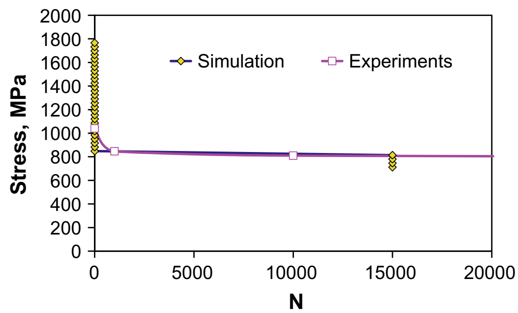

Fig. 12. S–N curves for a carbon/epoxy composite: experiments [36] and modeling.

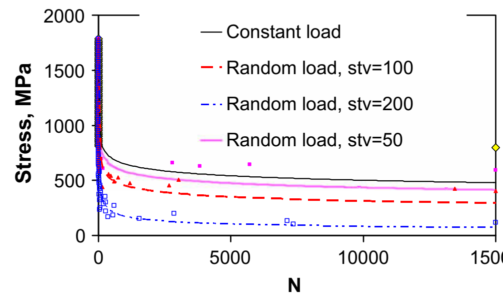

Fig. 13. Random compressive loading: S–N curves for the case of constant stress in each cycle, and for the cases of random loadings with standard deviations of 50, 100 and 200 MPa.

5. Computational experiments: cyclic loading

In this section, we seek to generalize the statistical model of the compressive strength of composites for the cyclic loading. The model is modified to take into account the truncated fiber misalignment distribution, variation of fiber misalignment and progressive matrix degradation. Then, the cyclic loadings with constant, random and increasing/decreasing loading are simulated, and the effect of the loading regime on the damage evolution is analyzed.

5.1. Modification of the model

The simplest way to generalize the static model above to the case of cyclic loading is to apply repeated static loadings. In order to compare the theoretical S–N curve with the experimental results, some additional corrections have to be introduced into the model. So, the ideal Gaussian distribution of fiber misalignments means that 2% of fibers have a misalignment more than five degrees. Apparently, this is not realistic, given the technology of fiber placement. Thus, we use the truncated normal distribution to describe the misalignment distribution in composite. In our further simulations, the distribution of fiber misalignments follows the normal probability law between -3^o and 3^o.

Fig. 12 shows the comparison of experimental [36] and theoretical S–N curves of carbon/epoxy composites. The 100 levels of applied stresses (constant in each cycle) were considered. For each cycle, a new unit cell (with 500 fibers and volume content 50%) was generated. Each cyclic loading simulation run up to 15000 cycles for each considered stress level. The simulation was stopped, if the damage in the composite exceeded 0.3.

It can be seen that the stress–lifetime curve, obtained in the simulations, does correspond well to the experimental results.

5.2. Random cyclic loading: effect of load variation on the damage evolution

The repeated constant loading is in fact a theoretical idealization. In reality, cyclic loading contain always some random components. In this subsection, the effect of the random variations of applied loadings on the damage evolution and fatigue life in the composite are studied.

The applied stress in each loading was random value. This value follows the Gaussian probability distribution law, and is determined with the use of the random number generator. Three levels of the standard deviation of the applied stress were considered: 50, 100 and 200 MPa. Fig. 13 shows the S–N curves (given as trend lines obtained for a number of points) for three levels of the stress variation, and for the case of the constant applied stress.

One can see that the random variations of applied loading lead to the shortening of the fatigue life of composite. For instance, the stresses, at which the lifetime of composite exceeds 15000 cycles, are 797 MPa for the case of constant stress in each cycle, 594, 406 and 117 MPa for the cases of the random loadings with standard deviations of 50, 100 and 200 MPa, respectively. Thus, even small random components of loading lead to rather large reductions of the lifetime.

Next, we consider the effect of the increasing and decreasing stress during the cyclic loading. We considered several cases: at each stress level, the applied stress is varied (linearly decreased or increased) during the cycling. The average stress over the cycling was the same in all the considered cases. The stress in j-th loading was determined by linear interpolation between the stress in the first loading of the cycling [which was given as $(1-q)^*\sigma_o$] and the stress in the last loading of cycle, given as $(1+q)^*\sigma_o$. Here, q – some value between -1 and 1, σ_o is the average stress in the cycle. The formula for determination of the applied stress in the i-th cycle was:

$$\sigma_i = 2q\sigma_o\left[\frac{j-1}{N-1}\right] + 1 - q, \tag{6}$$

If $q > 0$, the stress increases over the cycling, and if $q < 0$, the stress decreases.

Fig. 14 shows the stress corresponding to the lifetime 15000 cycles plotted as a function of the increasing/decreasing of the stress during cycling.

One can see that the increasing stress during the cycling leads to the higher lifetime at the high stresses, but lower lifetime at the low stress, as compared to the case of the constant stress loading. The decreasing stress during cycling results in that the composite the lower stresses ensure the lifetime.

5.3. Generalization for the low cycle fatigue and matrix degradation

As can be seen in Fig. 12, the S–N-curve obtained in the simulations is L-shaped: the material either fails during the first 10 cycles,

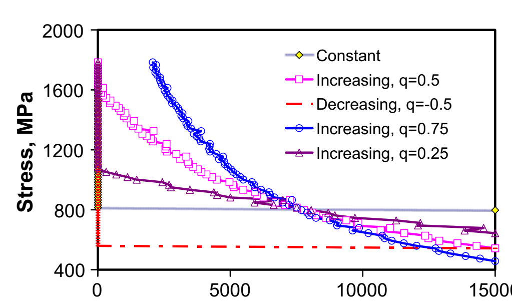

Fig. 14. Stress corresponding to the lifetime 15,000 cycles as a function of the increasing/decreasing of the stress during cycling.

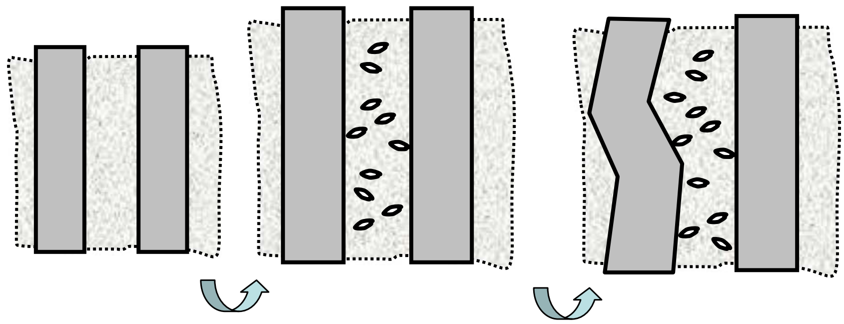

Fig. 15. Schema: Degradation of the composite matrix leading to the reduction of shear modulus of the matrix and to the fiber kinking.

or its lifetime is very high. In many cases, the S–N curves and especially the transition between the static failure and high cycle fatigue are much smoother.

In order to generalize our model for this case, some additional modifications have to be introduced into the model. Apparently, other damage mechanisms than the progressive fiber kinking (when a few fibers kink in each loading cycle) should be responsible for the smooth transition between the very low cycle fatigue and high cycle fatigue: as soon as the fiber kinking begins, the remaining lifetime is relatively short, due to the redistribution and increase of loading on remaining fibers.

We assume that the progressive damage in the matrix and/or on the fiber/matrix interface is responsible for the slow degradation of the composite properties during the undercritical loadings of the composite. This matrix damage leads ultimately to the begin of kinking and formation of the kink band. Fig. 15 shows the model schematically. In this model, the nanoscale degradation of polymer matrix due to the cyclic loading leads to the reduction of the shear modulus of matrix, and, ultimately, to the fiber kinking according to the Budiansky–Fleck condition [12] (for the fibers which do not kink according to the condition as long as the matrix is intact).

As a first approximation, the following degradation rule for the matrix was used:

$$G = G_0(1 - D_{matr})$$
$$D_{matr} = \sum_j \left(\frac{\sigma_a}{g}\right)^p \tag{7}$$

where D_{matr} – (anisotropic) damage parameter in the matrix, G_0 – shear modulus of the intact matrix, σ_a – applied stress, j – amount of cycles, g and p – fitting constants.

Fig. 16 shows the S–N curves for three cases: L-shaped S-N curve (no matrix degradation), and the curves for the cases of $p = 1$ and $g = 5 * 10^5$ and $g = 10^6$. The simulations were carried out up to 10,000 cycles. One can see that this model allows to obtain more realistic, smooth transition between low cycle fatigue and high cycle fatigue. However, the problems of the correct determination of the parameters p and q, as well as the improvement of the oversimplified model (7) remain unsolved, and represent a subject of our next study.

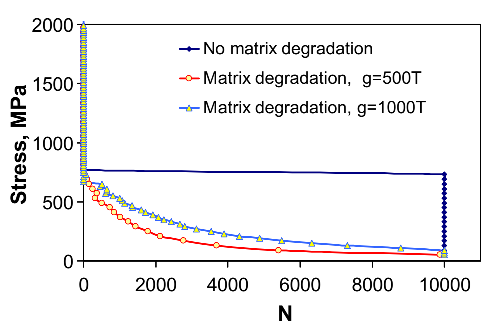

Fig. 16. Example S–N curves of composite: L-shaped S–N curve (no matrix degradation), and the curves for the cases of $p = 1$ and $g = 5 \times 10^5$ and $g = 10^6$.

5.4. Increasing of fiber misalignment at the repeated loadings

If the composite is subject to repeated compressive loading (even without by inflicting damage on the fibers and matrix) and some part of deformation is irreversible, it can lead to the changes of microstructure. Namely, the distribution of fiber misalignments can change. Even if a very small irreversible deformation of composite takes place, the misalignment angle can be increased by a few tenth of degrees after many cyclic loading. Given that the smallest variations of fiber misalignment play an important role in the kinking of the fibers, even such small growth of misalignment angles may influence the fatigue behaviour of composite.

Let us consider the effect of the compressive loading with irreversible deformation components on the fiber misalignment distribution. Fig. 17 shows a scheme of the increase of the fiber misalignment as a result of compressive loading, causing irreversible deformations. Next, we check that the variability (standard deviation) of their misalignment angles increases under such conditions as well (and not only the angle of each fiber misalignment). From the geometrical reasoning, the misalignment angle of each fiber after the fiber is subject to compressive load and irreversibly deformed is

$$\alpha_1 = arcsin[sin(\alpha_0) - \varepsilon]$$

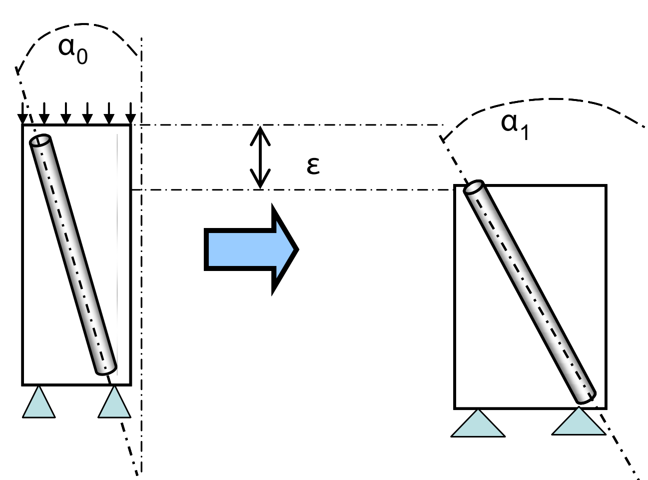

Fig. 17. Schema: Increasing the fiber misalignment as a result of repeated compressive loadings, causing irreversible deformations.

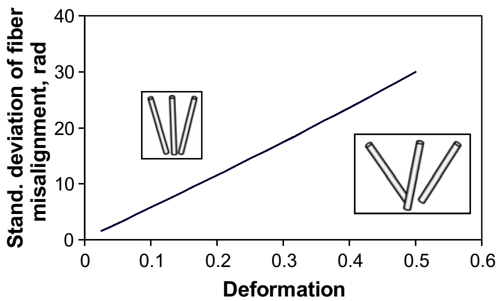

Fig. 18. Increase of the variability of fiber misalignment with increasing deformation of the cell.

where α_0 – initial misalignment angle, ε – irreversible deformation. We calculated the standard deviation of misalignment angles of an array of 100 fibers as a function of initial angles (standard deviation 1.125 degrees) and the irreversible deformation of the cell. Fig. 18 shows the increase of the variability (standard deviation of Gaussian distribution) of fiber misalignments with increasing irreversible deformation of the material. One can conclude that the variability of fiber misalignment can increase if the material is subject to repeated compressive loading, causing irreversible deformation of the composite. The effect of the changing misalignment distribution after repeated loading can influence the compressive fatigue behaviour of composites. The experimental verification of the results is under way now.

6. Conclusions

A statistical computational model of compressive loading of fiber reinforced composite with randomly distributed misalignments of fibers has been developed. Using this model, a series of computational experiments seeking to analyze the effect of composite microstructures on the compressive and fatigue strength was carried out. The following conclusions could be drawn from the simulations:

- The load sharing rule has a strong influence on the damage evolution in composites. The global load sharing leads to quicker damage growth. Thus, if we can localize the damage evolution (for instance, by placing weak fibers inside a group of strong fibers), we can delay the destruction of composite. Given the effect of the interface debonding on the load sharing [35], one can conclude that the availability and growth of interface debonding leads to the more homogeneous, ultimately global load distribution after fiber kinking, and, therefore, to the quicker damage growth.
- It was demonstrated that, the higher variability of fiber misalignments leads to the greater damage and stiffness loss in composites. An assumption is formulated that the variability of fiber misalignments can increase when the material is subject to repeated compressive loading, causing irreversible deformation of the composite.
- The clustered fiber arrangement leads to the failure of fibers under lower load. If fibers are clustered, failure of one fiber leads to the quick failure of all fibers in the cluster. For instance, at the compressive stress 1500 MPa, the damage in composite with clustered fibers is 32.5% higher than in the composite with homogeneously arranged fibers.
- The static compressive loading model was generalized to the case of cyclic loading, with constant, random and increasing/decreasing applied stresses in each cyclic loading case. It was shown that the random variations of applied loading lead to the shortening of the fatigue life of composite. Even small random components of loading lead to sufficient reduction of the lifetime.

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