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PROBABILISTIC STRENGTH ASSESSMENT OF FRP LAMINATES

Verification and comparison of analytical models

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Abstract: This document was prepared in the frame of Task 3.3 “Damage Tolerant Design Concept” of Work-Package WP3 “Rotor Structure and Materials” of the UPWIND project. Numerical procedures for determining the strength of a composite laminate, using various failure criteria, by taking into account the stochastic nature of anisotropic material properties are in detail described. The procedures were developed with the scope to form the base for methodologies that will be built up within the UPWIND project for quantifying the blade design reliability. Specifically, the Edgeworth Expansion Technique and the First Order Reliability Method are applied for the reliability estimation of composite material laminates. Results are compared with simulation predictions of the Monte Carlo method. Results by the Edgeworth Expansion Technique were found in good agreement with the respective ones from the other methods. Additionally, the Edgeworth Expansion Technique has a number of advantages compared to the First Order Reliability Method and the Monte Carlo simulation, since it does not involve iterative solutions for the reliability estimation. This makes the method attractive for application during the design of wind turbine rotor blades, where a probabilistic approach of the problem is expected to offer new potential in the direction of optimized material use.

Contents

STATUS, CONFIDENTIALITY AND ACCESSIBILITY							
Status			**Confidentiality**			**Accessibility**	
S0	Approved/Released		**R0**	General public		Private web site	
S1	Reviewed		**R1**	Restricted to project members		Public web site	
S2	Pending for review		**R2**	Restricted to European. Commission		Paper copy	
S3	Draft for commends	**X**	**R3**	Restricted to WP members + PL	**X**		
S4	Under preparation		**R4**	Restricted to Task members +WPL+PL			

PL: *Project leader* **WPL:** *Work package leader* **TL:** *Task leader*

1. Introduction

For the estimation of the reliability of a structure statistical or probabilistic calculus is employed. The probabilistic methods are usually applied to answer to two basic questions: What is the probability density function of the system response? Or alternatively and a little bit more concentrated on the actual problem: What is the probability the system response to surpass a specific critical value? From the structural analysis and the design based on reliability view point the second approach is more usual, since the engineers dealing with the analysis of structures are also responsible for the description of the probability of failure of the system relative to a group of design norms and specifications.

According to ISO 2394 [1] during the design the behavior of the whole structure or a part of it should be described in terms of a group of limit cases, out of which the structure does not satisfy the design specifications. The term failure is therefore, used with the general meaning of the word and states the shortcoming of the structure to satisfy a specific criterion of a limit state, which could be (or not) the global failure of the structure.

Having accepted the division of the structural behavior in a failure state and a safe state (or a state of no failure) we can then have a closer look at the methods which can be used for the estimation of the probability of each state. A reliability estimation method, in the strict sense is a method for the estimation of the failure probability, P_F, of a system. It is noted that reliability, P_R is the complementary probability of the failure probability. That is:

$P_R = 1 - P_F$

For the calculation of the failure probability a number of steps should be taken. First of all the basic variables of the system should be defined, such as the thermo-mechanical material properties, the geometry of the structure, the ambient conditions during operation (temperature, relative humidity) and the external loading conditions. The combination of the geometry and the material properties defines the resistance of the structure, or in the particular case studied the strength of the system. The variables describing the characteristics of the resistance/strength are called the design variables. Some of them (if not all) have a stochastic nature. The definition of the basic variables means, therefore, the selection of the probability density functions and the estimation of the statistical parameters of the distributions, in case they are characterized as stochastic, whether this is the result of a natural variation or the result of the incomplete knowledge we have of them. Both uncertainty sources, that is those of the external parameters and those of the internal are grouped into a generalized vector of basic variables, $\mathbf{X} = (x_1, x_2, \ldots x_N)$. The variations of those variables may be time dependent. In this study, however, the analysis is limited to those states, where the uncertainties can be defined independent of time. The response of the system under a particular value of the vector is denoted as $G(\mathbf{X})$.

After having defined the basic variables, a critical point is assumed to exist on the system response, resulting to the division of the system parameter space in two subspaces: The Ω space where the combination of the parameters leads to a non acceptable or a non safe response and the Ω' space where the system response is acceptable. The surface separating these two domains is called limit state surface or limit state function. The probability of failure of the system is defined by:

$$P_f = \iint \ldots \int_{\Omega} f_{\mathbf{X}}(\mathbf{X}) d\mathbf{X} \qquad \text{Eq. 1}$$

Where $f_X(\mathbf{X})$ is the joint probability density function. Apart from some simple failure functions and random variables following the Normal distribution, the integral of the joint probability density function is difficult to obtain analytically. This is also the case for the failure function of a composite material lamina under general in-plane state of stress. Thus, for the estimation of the failure probability approximating methods must be used. In the following sections some of those methods are described, with emphasis on their strengths and weaknesses on their application during the design of composite material structures.

1.1 Classification of reliability methods

The variety of idealizations of the reliability models and the multitude of ways that these idealizations can be combined in order to fit to a specific design problem makes the classification of these models necessary. Structural reliability models can be classified as to the extent of information available and used for the structural problem. Four such main levels could be identified, although this classification is not limited [2].

Level I: The reliability methods that use only one "characteristic" value for each parameter are called level 1. This classification includes the design methodologies that employ the characteristic values of the parameters and partial safety factors.

Level II: The methods that take advantage of two values for each parameter, usually the mean (average) value and the standard deviation and which include a measure of the correlation between the variables, usually the covariance, are called methods of level II. The reliability index methods, which will be presented in following sections, are classified under this category. These are structural reliability analysis methods which employ safety checks on a pre-selected point (or points) on the failure surface, that is, the proper limit function.

Level III: The reliability methods that use the failure probability as a measure and that require knowledge of the joint probability distribution of all stochastic parameters are called level III methods. These methods are purely based on the statistical analysis during which the safety checks are performed for the whole structural system by the simultaneous use of the probability distributions of the problem variables.

Level IV: Finally a reliability method that compares a candidate structural design with a reference structural design according to economic analysis principles from an engineering point of view, taking into account the uncertainties involved including the advantages and disadvantages of the manufacturing, the maintenance and the repair, the consequences of failure, the interest rates, etc. is called method of level IV. Such methods are suitable for structures that are of highest economic importance and the possibility of a life loss, injury and other environmental or not impacts should be kept to a minimum.

The type I approach is the best case for use by engineers for the structural design, but is not that advanced, so as to be applicable to all cases. The method does not require the exact estimation of the value of the failure probability, since a specific limit state is considered to be safe if the proper safety factors are not exceeded. A list of such safety factors is given in design codes for cases that have been already analysed. The problem is that the values of these safety factors for the requested reliability region of structural elements should be first made available through a reliability analysis performed either by a type II method or by a type III.

The type III method is in practice a pure mathematical method, which should be probably used for the analysis of special structures where the reliability level is of outmost importance or in cases where it is really necessary to optimize the structure.

The type II method is usually called reliability index method, the mathematics involved are rather simple and it is generally accepted that this method has the potential to be used either for the determination of the proper partial safety factors for the type I method or to be used directly during the design.

The classification for the type II and III methods can also be conducted based on the form of the underlying failure function, since for the reliability analysis of a structure the mathematical definition of the failure function is required as a first step. Moreover, the modelling of the failure function should be conducted in such a way so that the safe (and acceptable) operational region is clearly distinguished by the non acceptable region. Therefore, depending on the level of description applied for the modelling of the uncertainty the estimation methods can be classified into three categories: a) Random variables models, b) random process models and c) random field models [3].

1.1.1 Random variables models

The random variables models describe reliability problems, which are characterized by the failure function G(X), where G is the failure function and X the set of the basic random variables. The failure is defined by convention as the event [G(X) ≤ 0], while the event [G(X) ≥0] implies the safe state. The probability of failure is defined as:

$$P_F = P[G(X) \leq 0] = \int_{G(x)\leq 0} f_X(x)dx \qquad \text{Eq. 2}$$

Where $f_X(x)$ is the joint probability density function of the variables of X, which are independent of time.

Under these conditions, there are two basic categories of probability methods that can be applied. The first category includes a large group of random sampling methods, such as the Monte Carlo, or importance sampling techniques. These involve the random selection of observations of each parameter x_i of the system according to the probability function of the variable and these values are in turn inserted in the response function of the system G(**X**). The second category of reliability methods is characterised by the use of analytical techniques for the estimation of a specific point in the design space that can be correlated, at least approximately, to the probability of failure of the system. A review of such methods is given in [4].

In this work the analysis is limited to models of random variables, that is, it is considered that the random variables involved in the problem are independent of time and space. The methods that can be employed will be extensively described in latter sections of the current document.

1.1.2 Random process reliability models

In this case the problem is defined by the measure $\min_{t\in T} G(X, Y(t))$, where G is a failure function, X the set of random variables, Y(t) a set of random processes with t indicating the time and T a reference period, typically equal to the life of the structure. The probability of failure is in general the probability of first crossing. In the simplest case the failure surface in the Y space is deterministic and independent of time [3].

Basic principles of the problem are given in [5]. Random process models have been applied on composite material related problems by [6] among others, where the problem of composite material failure under a monotonically increasing random loading is examined. In another case, in [7], a deterministic time dependent problem is extended to account for the random applied loads (modelled as random processes) and the random material properties of an anisotropic viscoelastic medium to predict the initiation of delamination.

1.1.3 Random field reliability models

A random field reliability model is described by the function G(X, Y(t), Z(q, t)), where X is a set of random variables, Y(t) a set of time dependent random variables and Z(q, t) is a set of random variables dependent on the position on the field, q, where q could for example represent the coordinates of the position, and the time, t. Random fields are ideal for modelling variation in space, as in the case of material properties or the variation of distributed loads on a structure. An overview of the basic underlying principles of such problems is presented in [8]. Random fields have been used for modelling with stochastic finite elements the material properties of an element or for the nodal forces, as described in [9] among others.

Models of random fields, independent of time have been applied in composite materials related problems, especially for the study of their failure in the michromechanical level [10]. On the macromechanical level, applications of random fields are described in [11], where the strength of the material is modelled as a random field and in [12], where apart from the strength also the elastic material properties are described as random field variables. For both cases the prediction of the structural response is performed through the Monte Carlo method. Additionally, in [13] the elastic material properties and the material density are modelled as random fields in order to estimate the response of a composite material plate in free vibration.

A combination of random process and random field modelling in problems of composite materials is applied for example in [14], where the method applied in [6] is extended to account for the stochastic nature of the material elastic and strength properties, by use of random field models so as to predict the failure propagation up to ultimate fracture. Similar application of random processes and fields can be also found in [15].

2. Failure function

In the case of composite material structures and in particular for wind turbine rotor blades, for the modelling of the failure of the structural element under general in-plane static stress condition, the engineer faces a multitude of options regarding the expression of failure, which inevitably have a different grade of accuracy. From the various theories that have been developed over the years, some find wide acceptance and application during the design of structural elements, while for some others there is not yet experimental proof, or need a better understanding of the underlying problem. Even when considering the results and recommendations of the world-wide failure exercise [16] the designer is advised to implement two theories together to arrive at optimum results regarding failure prediction of unidirectional laminae under combined loading [16], while for the prediction of final strength of multidirectional laminates it was found that none of the 19 theoretical approaches considered in the failure exercise could claim very great accuracy [16]. In general the models regarding the failure of a composite material layer can be divided in micromechanical models, that is models that are used for the failure prediction of the layer starting on the fibre-matrix level and on macromechanical models, that are usually based on the failure description of the medium without getting into details on the fracture at the fibre-matrix level [17].

The failure phenomena that are developed on the microscopic level are rather complex, which in turn lead to models of equal complexity and of disputable accuracy and as a result are difficult to apply during the design of a structure [18]. Even though there has been a swift progress on the subject over the last years, the impression that models based on micromechanics are not suitable for incorporation on design procedures still exists [16]. A comprehensive review of the failure prediction methods for composite materials, based on micromechanics and damage evolution theories using probability theories is presented in [19]. A general conclusion of this rigorous review is that in many cases the developed theories are not verified through experimental results. This can be attributed to three main reasons: The wide field of materials in combination with different experimental techniques that are used for the comparison between theoretical and experimental results, the different manufacturing methods that can mask inherent trends of the results, as well as the difficulties in accurately determining the parameters used during the theoretical analysis, as for example the correlation length of a microcrack on the stress field. These three causes in combination with the lack of data for the composite material properties make the verification of a general theory rather difficult. Moreover, it is noted that since these estimation methods are mathematically mature, their application is usually cumbersome for the engineer. Further improvement of the methodologies will certainly lead to even more mathematically complex expressions, without necessarily present a solution to the problem of failure estimation [19].

On the other hand, considering the macromechanical approach of the problem, the failure criteria can be divided in parametric and phenomenological. The former are based on the approximation of experimental failure data through parametric equations, using geometrical criteria without constraints due to the phenomena that are taking place, such as the non-linear behaviour of the material [20]. The purpose of these methods is a more accurate modelling of experimental results, especially in cases where the use of phenomenological failure criteria fail. Nevertheless, such an approach requires a large number of test data covering the whole space of stress state to approach the main failure space accurately, while at the same time their application is not much simple than the other methods [21].

The phenomenological failure criteria are combining elements from the micromechanical failure analysis in a function (mathematical expression) that is applied for the failure prediction of composite material components rather straight forward. Consequently, based on the strength properties of the layer, however, without looking into the background of the failure mechanism, they find wide application in design problems of composite material structures, due to the fact that they are easily applicable, while having an acceptable level of experimental verification.

Nevertheless, also in this case, the differences between the various expressions of the failure conditions are several, since new forms of criteria are continuously developed, aiming at providing a sound explanation for the deviations of the past criteria on new experimental data. These criteria can be written as failure polynomial tensors [22] and include the maximum stress and maximum strain criteria. Especially these two criteria (maximum stress and maximum strain) fall in the failure conditions that neglect the stress (or strain) interaction terms. This fact in turn leads in some cases to the limitation of the criteria to effectively approach the failure state of composite materials. Nevertheless, both of them still find application, as for example in [23] where the failure of multilayered cylindrical specimens of $[90/\pm45/0]_s$ lamination sequence under internal pressure and axial tension is modelled. In most cases the failure criteria proposed are described in a quadratic form. Therefore, the maximum stress and maximum strain criteria present an additional drawback with respect to the ease of application, due to the third and fourth order terms that are incorporated in the failure function [22].

Consider the case of an off-axis unidirectional layer under general in-plane state of stress, as shown in Figure 1.

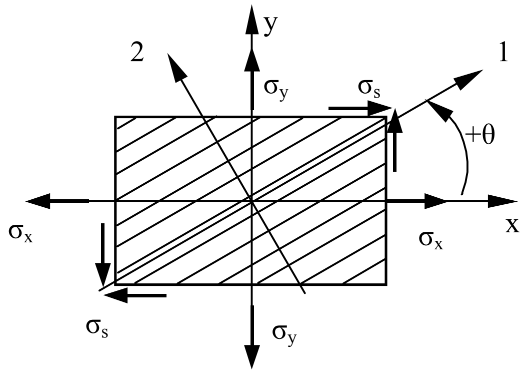

Figure 1 Coordinate system for an off-axis unidirectional layer

The failure tensor polynomial is used in the quadratic form, which in the natural coordinate system of the material, i.e. on-axis, under a general in-plane state of stress can be written as [24]:

$$H_{ij}\sigma_i\sigma_j + H_i\sigma_i - 1 \le 0 \qquad i, j = 1, 2, 6 \qquad \text{Eq. 3}$$

The diagonal terms of the failure tensors for a unidirectional (UD) fibre reinforced plastic (FRP) layer are given by:

$$H_{11} = (X_1X_2)^{-1} \qquad H_{22} = (X_3X_4)^{-1} \qquad H_{66} = X_5^{-2}$$
$$H_1 = X_1^{-1} - X_2^{-1} \qquad H_2 = X_3^{-1} - X_4^{-1} \qquad \text{Eq. 4}$$

where the failure stress in tension and compression along the fiber direction and transversely to it, X_T, X_C, Y_T and Y_C, respectively, as well as the in-plane shear strength, S, will be denoted for easiness by:

$$X_T = X_1, \quad X_C = X_2, \quad Y_T = X_3, \quad Y_C = X_4, \quad S = X_5 \qquad \text{Eq. 5}$$

The selection of the off-diagonal term, H_{12}, leads to different phenomenological failure theories, with predictions that may have important differences, [25], [26]. The failure criterion in the form proposed by Tsai and Hahn (TH) [27] uses the following definition for the H_{12} term:

$$H_{12} = -\frac{1}{2}\sqrt{H_{11}H_{22}} \qquad \text{Eq. 6}$$

Another, widely used expression for the off-diagonal term H_{12} is the following [28], [29]:

$$H_{12} = -\frac{H_{11}}{2} \qquad \text{Eq. 7}$$

In an effort to improve the failure predictions, where experimental results especially on lamination sequences around 45° could not be approximated effectively using quadratic failure criteria, led to the investigation of third order criteria, as for example in [30] and [31]. In this off-axis area the behavior of composite materials is usually non-linear. Thus, in [32] where the quadratic criterion was used with an off-diagonal term as in Eq. 7 and the non-linear material behavior was also taken into account the difference of the theoretical prediction from the experimental observations was substantially smaller.

In a rather different approach of the limit state problem of a composite material layer, where the phenomena in the micromechanical level were also taken under consideration, in [33] and later in [18] suggestions for failure criteria which can discern between fiber breakage and matrix failure were conducted. According to these criteria, for the investigation of the failure of a layer under general in-plane stress condition two equations should be checked simultaneously. It should be noted that these kind of criteria are also recommended in guidelines for composite material structural design, as for example in VDI 2014 [34], where the other quadratic failure criteria are considered as insufficient, since these cannot distinguish between fiber or matrix failure. This condition is also noted in design guidelines applicable on wind turbine rotor blades, e.g. GL [35] and DNV [36].

The variation of the experimental data which was observed, led to the estimation of the optimum angle between the fiber and the loading direction for the derivation of the coefficients that are applied on the failure functions, aiming at reducing the estimation uncertainty [37], [38]. On the other hand, the variation of the experimental results might lead to erroneous conclusions during the verification of the failure criteria [18].

Under these conditions, while the stress analysis for composite material structures is performed with the highest possible accuracy, the engineer encounters difficulties in answering the ultimate question of whether or not the structure fails under these loading conditions, even in the deterministic case, since there are a variety of failure criteria, with rather important differences. It is for this reason that in the current work an investigation has been undertaken for the effect of the selection of the failure criterion on the prediction of the failure probability of a composite material. Nevertheless, the sensitivity of the probability estimation using various failure criteria, was kept to a minimum, since the discussion on the most appropriate failure criterion for composite materials is still open [16]. The criterion which is used as the basis for the analysis is the quadratic failure condition Tsai-Hahn (TH), as expressed by Eq. 3, with the off-diagonal term given in Eq. 6, or alternatively the Elliptic Paraboloid Failure Surface criterion (EPFS) with the off-diagonal term as presented in Eq. 7, while the failure function in tensorial form with **X** the vector of the failure stresses ($\mathbf{X} = [X_1, X_2, \ldots, X_5]^T$), is expressed by:

$$K(\mathbf{X}, \sigma) = \sigma\mathbf{H}\sigma + \mathbf{h}\sigma - 1 \leq 0 \qquad \text{Eq. 8}$$

where it is assumed that if $K(\mathbf{X}, \sigma) < 0$ the layer is in a safe state, that is no failure is anticipated, if $K(\mathbf{X}, \sigma) = 0$ it is assumed that the layer is at a failure state, while the condition $K(\mathbf{X}, \sigma) > 0$ does not have a natural meaning since the failure of the layer has preceded.

For consistency with the definitions of the safe and failure state used in probabilistic methods the function G(**X**) is defined as:

$$G(\mathbf{X}) = -K(\mathbf{X}) \quad \text{Eq. 9}$$

3. Reliability estimation methods

3.1 Partial safety factors method

The design values of the basic variables are denoted by X_{id}. These values can be written as: $X_{id} = \gamma_i X_{ic}$, where X_{ic} is the characteristic value of the parameter X_i, while no summation is implied in this relation. The verification of the structure with respect to the limit state is performed inserting the design values and the dimensioning parameters in the failure function so as to confirm that:

$$G(\mathbf{X_d}) \geq 0$$

The characteristic values are usually the mean values, the 95% or 98% fractile for the loads and respectively the 5% or the 2% fractile for the strength properties. The partial safety factors are chosen, so as to be in the conservative side, that is, for example they are in general larger than unity for the loads and less than unity for the strength properties.

In the case of the wind turbine blade design, the respective design codes and standards, e.g. GL [35] and IEC [39], use is being made of the characteristic value of the failure stresses, R_k. The characteristic value of the failure stress is defined as a specific fractile, usually the 5% of the probability function of the relevant material strength, that is, the value of the strength z(a) defined by:

$$\int_{-\infty}^{z(a)} f(x)dx = F[z(a)] = a = 0.05 \qquad \text{Eq. 10}$$

where f(x) is the probability density function and F(x) the cumulative function of the underlying probability distribution which is followed by the failure stress of the material. It is usually assumed that the material strength follows the Normal distribution [40], [35], so that above relationship can be written as:

$$\int_{-\infty}^{z(a)} \phi(x)dx = \Phi[z(a)] = a = 0.05 \qquad \text{Eq. 11}$$

where $\phi(x)$ and $\Phi(x)$ are the probability density function and the cumulative distribution, respectively, of the Normal distribution. For the standardized variable $z = \frac{R_k - \overline{x}}{s}$, where $\overline{x}$ and s are the mean value and the standard deviation of the corresponding property, respectively, the 5% fractile is (see also Figure 2):

z(0.05) = -1.645

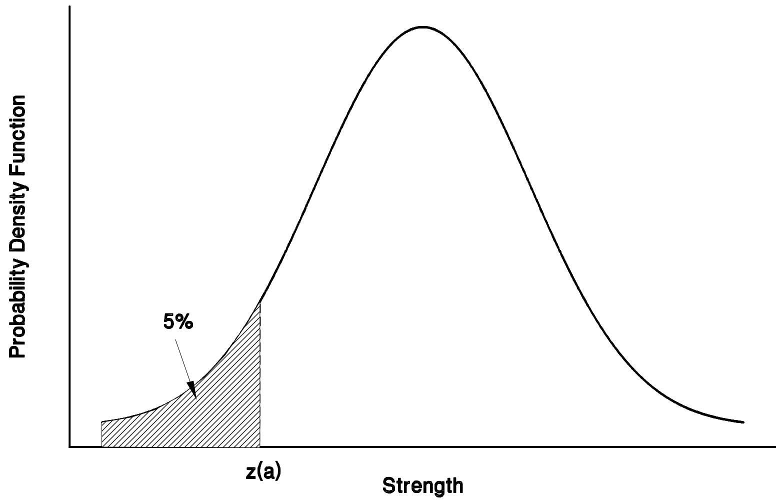

Figure 2 Definition of 5% fractile of the characteristic failure strength

Therefore,

$$R_k = \bar{x} - 1.645s \qquad \text{Eq. 12}$$

That means that the 95% of the population is expected to have a strength value larger than this characteristic value, R_k, which is derived by the solution of the above relationship.

However, to account for the fact that the mean value and the standard deviation of the population for the strength property have been estimated by a sample a confidence interval for an eventual lower mean value of the population than that found by the sample should also be used in the estimation of the characteristic strength value. The confidence interval selected is usually the 95% interval. The following relationship is valid for the lower bound of the mean value (assuming that the population follows Normal distribution):

$$\bar{x}_L = \bar{x} - t_{0.95}\frac{s}{\sqrt{n}} \qquad \text{Eq. 13}$$

where n is the number of specimens in the sample and $t_{0.95}$ is the Student distribution with v =n-1 degrees of freedom 95% fractile. For example the $t_{0.95}$ for 10 test specimens in the sample is 1.833, while for very large n, the Student distribution approaches the Normal distribution and $t_{0.95}$ is 1.645.

Thus, for example for a very large sample, the characteristic value, R_k, is determined through combination of Eq. 12 and Eq. 13 by following relationship, according to design codes specifications:

$$R_k(a, P, s, n) = R_k(5\%, 95\%, s, n) = \bar{x} - s\left(1.645 + \frac{t_{0.95}}{\sqrt{n}}\right) = \bar{x} - s\left(1.645 + \frac{1.645}{\sqrt{n}}\right) \qquad \text{Eq. 14}$$

where a is the requested fractile for the random strength property and P the confidence interval.

Frequently, in some design codes a partial safety factor can be written as:

$$\gamma_i = \gamma_{i1}, \gamma_{i2}, \ldots, \gamma_{iN}$$

where γ_{i1} is selected for example to account for the consequences of failure, γ_{i2} is selected to account for the uncertainty in the thermal treatment of the material, etc.

The characteristic values, the confidence intervals, the partial safety factors, etc., that is all values that are prescribed by a safety design code, apart from the mathematical or physical constants, form the parameters of the standard. The selection of a set of numerical values for the code parameters is called code calibration. Many of the design codes applied have been at least adjusted employing reliability estimation methods of level II, while it should be reminded that the classification of the partial safety factors method is level I [2].

3.2 Monte Carlo Simulation

The Monte Carlo simulation method is based on the iterative solution of the deterministic problem by generating for every repetition a probable value for each stochastic variable of the problem. In a problem involving stochastic variables, that follow a specific probability distribution, the Monte Carlo (MC) method comprises following steps:

- Sample generation for each basic stochastic variable according to the corresponding probability distribution of each variable.
- For each value of the samples solution of the deterministic problem and determination of the structural response.
- Formation of the response sample by the result of all deterministic iterations and finally,
- Statistical analysis of the response sample.

For the problem of the estimation of the reliability of a structure the analysis is simplified, since it is enough to check at each repetition whether the structural element fails or not. Thus, the reliability of the element is given by:

$$P_R = \frac{n_S}{n_{tot}} \qquad \text{Eq. 15}$$

where n_s is the number of iterations during the simulation where the failure function took values less than 0 and therefore, no failure was attained for the element and n_{tot} is the total number of iterations. A rule of thumb for the necessary iterations required so that the failure probability of the simulated structural systems is accurately predicted is $100/P_F$, where P_F is the expected probability of failure [3]. It should be noted that for the use of the MC method a scrutinized search has been performed in the frame of the current work not only for the number of iterations required for the convergence of the method, but also for the random number generation algorithms employed.

Due to the ease of application the MC method has been used several times in problems involving composite materials. For example, the MC method was the only used in [41] for the prediction of the failure probability of off-axis layers under uniaxial tension, where the strength properties have been modelled as stochastic parameters, while in [42] the MC method was applied for the estimation of the failure probability of every layer in a laminated plate under transverse load. In this latter case apart from the strength properties also the elastic properties, as well as the basic dimensions of the plate where considered as stochastic. Nevertheless, due to the high numerical effort, the MC method is not suitable for application on structures of complicated geometry, where the finite element model comprises a large number of elements. However, the method remains a valuable asset for the verification of methods newly developed for the prediction of reliability.

3.2.1 Statistical tests for the MC method verification

Since a sample of values (responses) produced through the MC simulation is similar to a sample of experimental observations, the method could be regarded as a sampling method and therefore, the results are subject to statistical uncertainties. A solution with the MC method from a finite set of values is not accurate. Specifically, the estimated probability, which is the desirable output, could differ from simulation to simulation by changing only the input samples of the basic variables. The first step of the method, which is also the most important one with respect to the accuracy of the output, is based on a procedure by which series of numbers are generated. The procedure involves the use of a pseudo-random number generator. In this work the subroutines used for the generation of the pseudo-random numbers are described in [43]. Moreover, the sample used during the simulation could be adequate for a specific application, but could be insufficient for another. For the verification of the suitability of the number generators there are several statistical tests. The good random number generators should pass all these tests, otherwise the user should be aware of in which of these tests the generator is rejected [43]. It should be also noted, that a specific algorithm should be tested together with the pseudo-number generator employed independently of the number of tests that the generator passes [44].

Since the MC method is used in the current work as a base for the comparison of results attained by other methods an extensive verification of the soundness of the method application has been performed. To this, it is noted that generally with respect to the verification of the MC method there are two views in the literature; that of the mathematicians [45], [44], [46], [47] and that of the engineers [48] and [3]. The former perform tests starting from the base of the method, that is from the method of random numbers generation, while the latter use the method, often omitting to test the samples that are being used as an input during the simulation and are restrained to verifying the convergence and the accuracy of the prediction.

To be more specific, for the verification of the application of the simulation from the mathematical point of view it is necessary to test the employed algorithm starting from the random number generator to the final output. To this end, not only theoretical tests for the soundness of the generator are necessary, but also empirical tests for the samples of the pseudo-random numbers that are generated, tests for the transformation of samples used from the uniform distribution to the distribution of interest and finally, tests for the application of the method. On the contrary, from the engineering point of view, only a test for the convergence of the method is necessary, namely, the good choice of the sample size and the minimization as far as possible of the standard deviation of the prediction.

Apart from that, during the applications, as these encountered in the current work, it is rare to need a sample generation for only one variable. Usually the samples of a generator are split into subgroups, so as to form samples for more than one variable. This procedure, however, results in the alteration of the behaviour of the generator sample regarding randomness [44]. Thus, to correctly confront the problem, after selecting a good generator from a theoretical point of view, a test on the samples produced during the application should be conducted, which in turn should be followed by the verification of the convergence and the accuracy of the prediction of the specific problem.

For the current work statistical tests have been performed for two different cases. In the first case the behaviour of the generator has been tested with respect to the samples generated and used during the application. As a second case statistical tests have been performed to evaluate the appropriateness of the samples from a uniform distribution after their transformation to samples following a different statistical distribution for the simulation of the material properties of the layer. This second case was evaluated basically to assure that the transformation of the samples from one distribution to the other does not affect the final result.

In the first part, for the empirical testing of the generators following tests have been performed:

- Kolmogorov-Smirnov Test
- Gap Test
- Serial correlation test
- Up and Down Test
- Maximum of t Test
- Permutation Test

The scope of all tests performed was to verify the correct use of the MC method in the current work. As far as the empirical tests concerned, these have been selected as the more characteristic ones within the literature [44], [46]. In the second part, where the purpose was to verify the correctness after the samples' transformation from the uniform distribution to some other, the Kolmogorov-Smirnov Test has been employed.

It should be noted, that all statistical tests performed, where applied in combination with the sample generations algorithms employed in the applications of the current work, analysing this way, not only the adequacy of the random number generators selected, but also all subgroups of the generated samples.

3.3 Mean Value Method

Usually the information at hand or the data are sufficient only for the estimation of the first and second moment, namely the mean value and the variance of a random variable. Therefore, for the estimation of the reliability of a structural system the functionals of the methods employed should be limited to function of these first two moments. Under these conditions the reliability can be estimated using a function of the first and second moments of the design variables, when no information is available for the underlying statistical distributions.

Consequently, use of this information for the basic variables can lead only to an estimation of the mean value and the variance of the failure function, applying a Taylor expansion on the function about its mean value. For the method of generation of system moments [49] (also called statistical error propagation method or delta method), let a function z = z(**X**) with **X** the vector of the N basic variables. Then, applying the Taylor expansion of z of the N variables around $\mathbf{X_m}$, $\mathbf{X}_m^T = [X_{1m},...,X_{Nm}]$, where X_{im} is the mean value of variable X_i and neglecting terms higher than 2nd order it is:

$$z(\mathbf{X}) \cong z(\mathbf{X}_m) + z_i(\mathbf{X}_m)\overline{X}_i + \frac{1}{2} z_i^{(2)}(\mathbf{X}_m)\overline{X}_i^2 + z_{ij}^{(2)}(\mathbf{X}_m)\overline{X}_i\overline{X}_j \qquad \text{Eq. 16}$$

where the following notation has been used:

$$\overline{\mathbf{X}} = \mathbf{X} - \mathbf{X}_m \qquad \text{Eq. 17}$$

while in Eq. 16 summation of terms with repeated subscripts is implied, and the partial derivatives of function z(**X**) with respect to the variables X_i are denoted by:

$$z_i(\mathbf{X}) = \frac{\partial z(\mathbf{X})}{\partial X_i} \qquad z_i^{(2)}(\mathbf{X}) = \frac{\partial^2 z(\mathbf{X})}{\partial X_i^2} \qquad z_{ij}^{(2)}(\mathbf{X}) = \frac{\partial^2 z(\mathbf{X})}{\partial X_i \partial X_j} \qquad \text{Eq. 18}$$

The method of moments can be used for the estimation of the central moments of z(**X**) in terms of the corresponding moments of the variables X_i, which are defined by:

$$\mu_{k_i} = \mu_k(X_i) = \int_{-\infty}^{\infty} (X_i - X_{im})^k f(X_i) dX_i \qquad k = 2, 3, 4 \qquad \text{Eq. 19}$$

where $f(X_i)$ denotes the probability distribution function (PDF) of variable X_i. The mean value of the function $z(\mathbf{X})$ in case the basic variables of the vector **X** are uncorrelated is given by [49]:

$$E[z] = z(\mathbf{X}_m) + \frac{1}{2} z_i^{(2)}(\mathbf{X}_m)\mu_{2i} \qquad \text{Eq. 20}$$

The variance of the function $z(\mathbf{X})$, respectively, when the basic variables of the vector **X** are uncorrelated is given by [49]:

$$\mu_2[z] = (z_i(\mathbf{X}_m))^2 \mu_{2i} + z_i(\mathbf{X}_m) z_i^{(2)}(\mathbf{X}_m)\mu_{3i} \qquad \text{Eq. 21}$$

For completeness, also the third and fourth moment of the function $z(\mathbf{X})$, when the basic variables of the vector **X** are uncorrelated is given by [49]:

$$\mu_3[z] = (z_i(\mathbf{X}_m))^3 \mu_{3i} \qquad \text{Eq. 22}$$

$$\mu_4[z] = (z_i(\mathbf{X}_m))^4 \mu_{4i} + 6\left(z_i(\mathbf{X}_m) z_j(\mathbf{X}_m)\right)^2 \mu_{2i}\mu_{2j} \qquad \mathbf{i < j} \qquad \text{Eq. 23}$$

It should be noted that the method of moment generation is approximate, while its accuracy depends on the specific function $z(\mathbf{X})$. Keeping terms up to the second order improves the accuracy for non linear functions. However, the accuracy of the estimations also depends on the quality of the statistical information of the basic variables. If experimental data from small samples are used the uncertainty inserted in the estimation of the system moments could be much greater than that neglecting the higher order terms [49]. In the applications usually presented, the estimation of the first two moments of the function are performed using only the first order terms, reducing the accuracy for non linear functions. In the current work the estimation of the moments is performed by retaining also the second order terms.

In the simplest case, the estimation of the failure prediction is performed based on these two moments and usually assuming that the function follows the Normal distribution. In [4] it is mentioned that the basic causes of error relating to the mean value method is precisely the assumption that the response follows the normal distribution in combination with the Taylor expansion as an approximation of the function. Moreover, it is noted that the results of any uncertainty analysis method should be independent of the mathematical expression of the failure function of the problem, when the initial definition of failure is kept. To be more specific, if we assume that the failure function of a problem is defined by $G = R – S$, the results of the analysis for the probability of failure $P_F = P(R<S)$ should be equal to that of the probability $P_F = P(R-S<0)$ or of the probability $P_F = P(R/S<1)$. This condition, however, is not satisfied for all cases in the mean value method. For example, considering the following equivalent expressions for the safe state of the Tsai-Hahn failure criterion for composite materials:

$$K = \frac{\sigma_1^2}{X_T X_C} + \frac{\sigma_2^2}{Y_T Y_C} + \frac{\sigma_6^2}{S^2} + \sigma_1\left(\frac{1}{X_T} - \frac{1}{X_C}\right) + \sigma_2\left(\frac{1}{Y_T} - \frac{1}{Y_C}\right) - \frac{\sigma_1\sigma_2}{\sqrt{X_T X_C Y_T Y_C}} \le 1$$

and

$$\begin{aligned} K = {} & \sigma_1^2 Y_T Y_C S^2 + \sigma_2^2 X_T X_C S^2 + \sigma_6^2 X_T X_C Y_T Y_C + \sigma_1 Y_T Y_C S^2 (X_C - X_T) + \\ & \sigma_2 X_T X_C S^2 (Y_C - Y_T) - \sigma_1\sigma_2 S^2 \sqrt{X_T X_C Y_T Y_C} - X_T X_C Y_T Y_C S^2 \le 0 \end{aligned}$$

with X_T, X_C, Y_T, Y_C the strength properties along the fiber direction and transversely to it in tension and compression, respectively and S the in-plane shear strength and σ_1 and σ_2 the applied stresses along the fiber direction and transversely to it respectively and σ_6 the applied in-plane shear stress, the results for the probability of failure by use of the mean value method are different.

Nevertheless, application of the method in combination with the finite element method can be found in [50] for the evaluation of the first ply failure probability and the buckling of a laminate. In this application stochastic parameters are considered the elastic material properties, the orientation of the fibers, as well as the thickness of the layers. Comparing the results obtained through application of this method with the results of the Monte Carlo simulation the accuracy achieved is acceptable.

In case more information are available for the basic variables of the problem, as for example the statistical distribution that these variables follow or the higher order central moments, these additional data can be used in order to improve the accuracy of the estimation of the failure probability. In [51] a method is presented according to which, if the central moments of the failure function are known, then the cumulative distribution function of the failure function can be approximated through use of the Lamda family distributions. Similar to the approach of [51] the Edgeworth Expansion technique and the Pearson family distributions can be used as described in the following sections.

3.3.1 Edgeworth Expansion Method

According to the Edgeworth Expansion Method an unknown cumulative distribution function of variable x can be approached by use of a series expansion of the Normal distribution, Φ(x) in terms of the central moments of x. This is given by [52]:

$$\begin{aligned} F(x) &= \Phi(x) \\ &-\frac{1}{3!}\frac{\mu_3}{\mu_2^{3/2}}\Phi^{(3)}(x)+ \\ &+\frac{1}{4!}\left(\frac{\mu_4}{\mu_2^2}-3\right)\Phi^{(4)}(x)+\frac{10}{6!}\left(\frac{\mu_3}{\mu_2^{3/2}}\right)^2\Phi^{(6)}(x)+\ldots \end{aligned} \qquad \text{Eq. 24}$$

where $\Phi^{(n)}(x)$ are the n-th order derivative of the Normal distribution function.

Although the approach is asymptotic, according to [52] it is not advisable to use terms containing higher moments than the third, or fourth, since these are difficult to obtain by use of small sample sizes. Moreover, the salient point is to approach effectively the unknown cumulative distribution function with only a few terms. Thus, the approach with two terms of the series for the failure function, K, in terms of the standardized variable z is given by:

$$F(a) = P(K \le a) \approx \Phi(z) - \frac{1}{3!}\frac{\mu_3(K)}{(\mu_2(K))^{3/2}}\Phi^{(3)}(z) \qquad \text{Eq. 25}$$

where:

$$\Phi(z) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{z} e^{-s^2/2}ds \qquad \text{Eq. 26}$$

$$z = \frac{a - E[K]}{(\mu_2(K))^{1/2}} \qquad \text{Eq. 27}$$

It should be noted that for large K the expression of Eq. 25 can sometimes lead to negative values of F(a), but this is the approximate nature of the probability distribution, since the expression does not result in the exact value of the distribution [52]. Moreover, in the case of the failure function, we are not interested in the exact value of the function, but rather in the probability of survival, that is, the probability that K has a value less than zero, $P(K \leq 0)$. Conversely, the probability of failure, P_F, is expressed by the relationship $P(K > 0) = 1 - P(K \leq 0)$, since the two events are mutually exclusive.

Additionally, due to the fact that the estimations for the 3rd and 4th order central moments are sensitive to observed outliers, the method should be used with care, especially if a small sample size has been used (less than 200 observations) [49]. The accuracy of the approximation should be checked with proper statistical tests, as for example the Kolmogorov-Smirnov test.

The Edgeworth Expansion technique has been initially applied in [53] for the approximation of the failure probability of an off-axis composite material layer under uniaxial loading conditions, employing the Hill failure criterion as the failure function. The Edgeworth Expansion method is also used in the current work and was further developed for application in the design of composite material wind turbine blades.

In comparison to the mean value method, the Edgeworth Expansion at first view presents several similarities. The main differences of these two methods are two: The first is that the first two moments of the failure function for the Edgeworth Expansion application is performed by keeping terms up to the second order, increasing this way the accuracy of the method for non-linear functions. The second is that following the mean value method it is assumed that the failure function follows the Normal distribution, while for the Edgeworth Expansion method the estimation is improved by using also higher order moments of the failure function.

The condition requiring the probability estimation method to be independent of the expression of the failure function is not satisfied also for the Edgeworth Expansion method for many problems as for the case of the mean value method. The point is however, that the method as applied in the problems that will be presented in following sections of the current document, is proved sufficiently accurate taking into account the small computational cost of the method in comparison to the others used in the current work.

As far as the number of terms in the expansion that should be kept for the present method in order to estimate the cumulative distribution function of the failure function, as shown in [54] use of two terms is sufficient.

3.4 First order reliability method

The first order reliability methods (FORM) can be characterised as extensions of the mean value method, developed to compensate for several technical difficulties related to the application of this basic method [4]. The main difference of the FORM in comparison with the mean value method is that the Taylor expansion of the failure function is not performed around the mean value of the function, but at a different point, which is called most probable failure point, as described in the following.

3.4.1 Hasofer-Lind method

The first step in order to satisfy the condition for the invariance of the reliability estimation method independently of the mathematical expression of the failure function was the transformation of the random variables of the problem to a set of normalized uncorrelated random variables through an orthonormal transformation [55], [4]. To this end, apart from the transformation of random variables following the Normal or the Log-normal distribution there several exact or approximate transformations [56], [57] have been proposed for variables following other distributions. The most widely used of those is the Rosenblatt transformation [58]. The failure function is thus, defined in terms of the normalized and independent variables.

This approach is independent of the statistical distributions of the random variables, since of the Rosenblatt transformation only the first two moments (the mean value and the variance) of the variables are needed. The estimation of the safety index and consequently a first estimation of the failure probability can be performed, this way, irrespective of the information of the statistical distribution of the problem variables. In case there is a need to improve the accuracy of the response estimation in probabilistic terms, then the information of the statistical distributions of the basic variables can be integrated in the solution of the problem, as described in the following section of the current document.

Considering the set of independent normalized variables:

$$X'_i = \frac{X_i - m_{X_i}}{s_{X_i}} \qquad i = 1, 2, \ldots, N \qquad \text{Eq. 28}$$

where $m_{X_i} = E[X_i]$ the mean value and $s_{X_i} = \sqrt{\mu_2(X_i)}$ the standard deviation of variable X_i the safe state and the failure state, which are separated by the limit state function, can be represented in the space of the above reduced variables. In terms of the reduced variables, X'_i, the limit state function can be written as:

$$g\left(s_{X_1}X'_1 + m_{X_1}, s_{X_2}X'_2 + m_{X_2}, \ldots, s_{X_N}X'_N + m_{X_N}\right) = 0 \qquad \text{Eq. 29}$$

For example in Figure 3 the limit state function for an on-axis unidirectional composite material is presented in the space of the random variable X and the reduced variable X'. The criterion used is the failure polynomial tensor with the non-diagonal term $H_{12} = -\frac{H_{11}}{2}$, which is written in terms of the random variables of the problem as:

$$G(\mathbf{X}) = -K(\mathbf{X}) = -\frac{\sigma_1^2}{X_T X_C} - \frac{\sigma_2^2}{Y_T Y_C} - \frac{\sigma_6^2}{S^2} - \sigma_1\left(\frac{1}{X_T} - \frac{1}{X_C}\right) - \sigma_2\left(\frac{1}{Y_T} - \frac{1}{Y_C}\right) + \frac{\sigma_1\sigma_2}{X_T X_C} + 1 = 0$$

Moreover, for the presentation in the space of two variables, it is assumed that only the strength in tension along the fiber direction and transversely to it, X_T and Y_T are random variables, while the applied shear stress is zero.

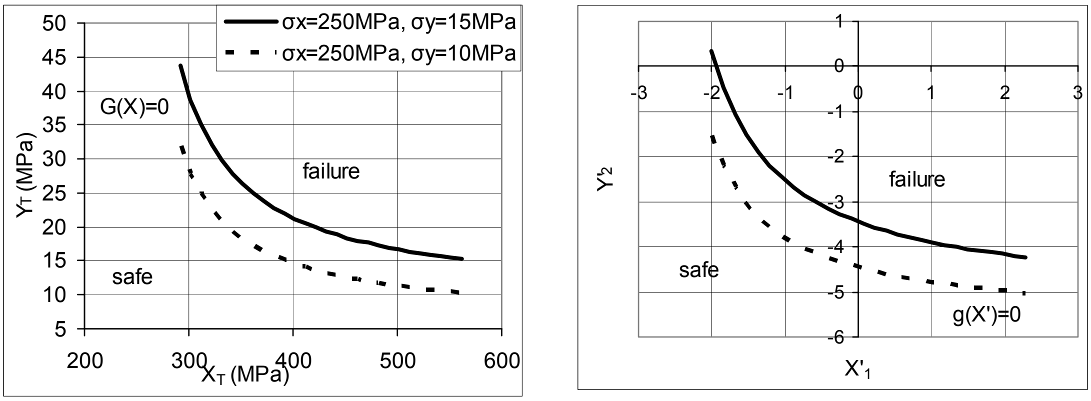

Figure 3 Example of limit state on the space of the original variables of the problem (left) and the respective reduced variables (right)

In Figure 3 one should note that the surface of the limit state $g(\mathbf{X'})=0$ approaches or departs from the origin as the safe state reduces or enlarges, respectively (on the specific example, enlargement of the safe state is accomplished by reducing the applied stress transverse to the fiber direction, $\sigma_y = \sigma_2$). Consequently, the position of the failure surface with respect to the origin in the space of the reduced variables is indicative of the reliability or safety of the system. The position of the failure locus, which in the space of two variables is a curved line, could be defined through the minimum distance of the surface $g(\mathbf{X'})=0$ from the origin in the space of the reduced basic variables of the system.

It has been proven [59] that the point on the failure locus, which defines the minimum distance of the limit state surface from the origin in the space of the reduced variables, is the most probable failure point.

Thus, in an approximating sense, this minimum distance form the origin can be used as a measure of reliability [48]. The requested minimum distance from a point $\mathbf{X'}=(X'_1, X'_2, ..., X'_N)$ on the failure surface $g(\mathbf{X'})=0$ to the origin of the $\mathbf{X'}$ space is given by [59]:

$$D=\sqrt{X_1'^2+X_2'^2+...+X_N'^2}=\left(\mathbf{X'}^T\mathbf{X'}\right)^{1/2} \qquad \text{Eq. 30}$$

The point on the failure surface $(x_1'^*, x_2'^*, ..., x_N'^*)$, which has the minimum distance from the origin can be found by minimizing function D, under the constraint $g(\mathbf{X'})=0$ using for example the method of Lagrange multipliers [48]. The minimum distance to the origin is called the reliability index, β, while the failure probability is defined by:

$$P_f \approx \Phi(-\beta)$$

Where $\Phi(x)$ is the cumulative probability of the Normal distribution.

It should be noted that if equation $g(\mathbf{X'})$ is linear with respect to the normalized variables, then the above solution would be exact, while in Figure 3 the failure function would be presented by a straight line.

The method found application in problems involving composite materials during the 1990s, while one of the pioneering works for the reliability estimation of a laminate is [60], where the applied loads or to be more specific the applied layer stresses are considered as random variables.

3.4.2 Rackwitz-Fiessler Method

With the Rackwitz-Fiessler method an improvement to the method is offered to account for basic random variables following other than the Normal distribution. As a result, the failure probability estimation is performed more accurately. To be more specific, if the statistical distribution of the random variables, $X_1, X_2, \ldots, X_N$, is not the Normal distribution, the probability of failure, P_F, or the probability of survival, P_S, can be estimated using an equivalent to the Normal distribution. Theoretically, such an equivalent distribution can be obtained by use of the Rosenblatt transformation, as shown in [56]. With employment of the equivalent distribution the calculations for P_S follow the same procedure as in the case where the variables are Normal variates.

For a variable that is not a Normal variate, the equivalent Normal distribution can be approximated so that the cumulative distribution and the probability density of the equivalent Normal distribution are equal to that of the non-normal distribution at the appropriate point x_i^* on the failure surface [48]. In mathematical terms, this can be written as:

$$\Phi\left(\frac{x_i^* - m_{X_i}^N}{s_{X_i}^N}\right) = F_{X_i}\left(x_i^*\right)$$

$$\frac{1}{\sigma_{X_i}^N}\phi\left(\frac{x_i^* - m_{X_i}^N}{s_{X_i}^N}\right) = f_{X_i}\left(x_i^*\right) \qquad \text{Eq. 31}$$

όπου $m_{X_i}^N$ and $s_{X_i}^N$ is the mean value and the standard deviation, respectively of the equivalent Normal distribution for X_i, $F_{X_i}\left(x_i^*\right)$ is the original cumulative distribution function of X_i and $\Phi(-)$ is the cumulative distribution of the standard Normal distribution, while $f_{X_i}\left(x_i^*\right)$ and $\phi(-)$ are the corresponding probability density functions. Solving the above system of equations for the unknown mean value and standard deviation of the equivalent Normal distribution for X_i at x_i^* one has:

$$m_{X_i}^N = x_i^* - s_{X_i}^N \Phi^{-1}\left[F_{X_i}\left(x_i^*\right)\right] \qquad \text{Eq. 32}$$

$$s_{X_i}^N = \frac{\phi\left\{\Phi^{-1}\left[F_{X_i}\left(x_i^*\right)\right]\right\}}{f_{X_i}\left(x_i^*\right)} \qquad \text{Eq. 33}$$

Thus, the variable X_i at the failure point x_i^* is normalized by:

$$x_i'^* = \frac{x_i^* - m_{X_i}^N}{s_{X_i}^N} \qquad \text{Eq. 34}$$

During the iterative approaching procedure of the most probable failure point, that is the point on the failure surface $g(\mathbf{X}') = 0$, which is at the smallest distance form the origin, the mean value and the standard deviation of the equivalent Normal distribution should be calculated at each iteration using the relations of Eq. 32 and Eq. 33. Thus, for the normalization of the variable at every iteration the true mean value and standard deviation of the variable are replaced with the corresponding ones of the equivalent Normal distribution [48]. The algorithm for the iterative procedure followed for the estimation of the most probable failure point and the β index is presented in [56]. The procedure includes the transformation of the original non-normal random variables, which might also be correlated, to independent variables following the Normal distribution.

The method, as described in the above, from now on called FORM, was applied in [61] for the reliability estimation of a composite material layer under uniaxial loading, considering the strength properties and the off-axis angle of the UD layer in combination with the applied stresses as stochastic variables. Moreover, a comparison is presented in the same work with results obtained applying the Hasofer-Lind method, concluding that the latter results in less conservative estimations than FORM. Additionally, FORM is applied in [62] for the last ply failure probability estimation of a multilayered plate under transverse loading, where again the strength properties of the material and the transverse load were assumed to be the basic variables of the problem. The results of the FORM method were found in good agreement with that of a MC simulation. Furthermore, in [63] FORM method is used for the first ply failure estimation of a multilayered plate under transverse loading in combination with the finite element method. In this case, however, only the material strength properties are considered to as random variables.

3.4.3 Higher order reliability methods

For non-linear failure functions, as in the case of the failure function of a composite material layer, the exact calculation of the failure probability or the reliability generally involves mathematical and computational difficulties. In contrast to the linear case, there is not always a single solution for the minimum distance of the failure surface to the origin in the space of the reduced variables. However, for practical cases it is necessary to approximate the failure probability of interest, assuming that the point $\mathbf{X}'^*$ on the failure surface having the minimum distance form the origin on the space of the reduced variables is the most probable failure point. The tangent plane on the failure surface at the point $\mathbf{X}'^*$ can then be used for the approximation of the true failure surface and the required reliability or safety probability can in turn be calculated as in the case of having a linear failure function [48]. Depending on whether the true non-linear failure surface is concave or convex with respect to the origin, this approximation could be conservative or non conservative, respectively, as presented in Figure 4 for the case of two basic variables.

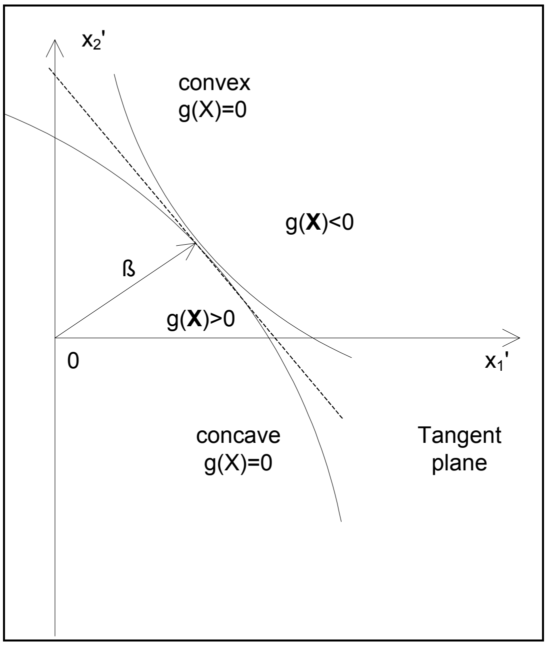

Figure 4 Non-linear failure functions on the reduced variables space

The required tangent plane at the point $\mathbf{X}'^*$ is defined by:

$$\sum_{i=1}^{N}\left(X'_i - x'^*_i\right)\left(\frac{\partial g}{\partial X'_i}\right)_* = 0 \qquad \text{Eq. 35}$$

Where the partial derivatives of $g(\mathbf{X})$ are calculated at the point $\mathbf{X}'^*$.

Based on the above approach, the minimum distance from the tangent plane to the origin defines the appropriate reliability index, which can then be used as a measure of the reliability [48].

In general, the linear approximation of non-linear failure functions is equivalent to replacing the N-dimensional failure surface (hyper-surface) with a hyper-plane, which is tangent to the failure surface at the most probable failure point. Usually, the accuracy of the linear approximation of the second moment is difficult to estimate and this depends on the degree of non-linearity of the function $g(\mathbf{X})$. For a general non-linear $g(\mathbf{X})$, the accuracy can only be estimated numerically for specific forms of failure functions [48]. The accuracy can be improved by considering a quadratic or a polynomial approximation of higher order, which, however, effectively leads to a higher numerical/computational cost [64].

For the reliability estimation of a lamina a quadratic approximation has been applied in [65], while for the reliability estimation of a laminate the quadratic approximation has been implemented in [66]. Nevertheless, in both cases it was found that the results are close to estimations conducted using the FORM method, making the higher numerical cost as a result of the quadratic approximation unjustifiable.

3.4.4 Algorithms for the estimation of β index

Several algorithms have been proposed for the approximation of the most probable failure point and the β index, e.g. [48], [4] and [2]. A comparison between five algorithms that can be used for the approximation of the most probable failure point has been conducted in [67]. The general conclusion of that work is that the selection of the more effective algorithm depends on the failure function of interest. Similarly, in [68] a new search algorithm is proposed for the estimation of the β index and its application is again compared with other widely used.

One of the main reasons for investigating alternative reliability estimation methods within the frame of the current work is that application of the β index method comprises an iterative procedure. Moreover, since it is not certain that the procedure will converge for all cases examined the method is unattractive for application during the design phase of a wind turbine blade, which is of interest.

Furthermore, since the β index method is used in the current work for comparison purposes, it was not considered necessary to investigate the application of the various algorithms presented in the literature. Therefore, the algorithm that is has been adopted is the one described in [2], which is rather widely used. It should be noted, however, that there is no guarantee that the algorithm converges in all cases, while it is probable during its application to result in values for the basic variables that are out of their natural limits [2].

4. Lamina failure probability

4.1 Lamina failure probability under uniaxial load

For the investigation of the strengths and weaknesses of the reliability estimation methods in composite material structures in general, as a first step the reliability of a lamina in uniaxial stress state is studied. The effectiveness of the various methods presented is assessed comparing the respective results with that of Monte Carlo simulation and experimental data, wherever these are available. For a first application the case of off-axis unidirectional layers is investigated.

The use of the EDW method for the estimation of the failure probability of off-axis coupons in tension in conjunction with the Hill failure criterion [69] was initially presented by Wetherhold in [53]. The method has been applied for the case of a unidirectional off-axis Glass/Epoxy layer under uniaxial tension load for comparison with experimental data from [41]. Moreover, in [53] it was shown that the results using either the maximum stress or the maximum strain failure criterion were not in good agreement with the experimental data. Nevertheless, in [54] it was shown that the EDW method results in better estimates when used in conjunction with the quadratic failure criterion. Due to this reason and taking into account the remarks presented in section 2 only the quadratic form of the failure criterion is applied in the current work. In [54] apart from results from application of the Edgeworth Expansion Technique (EDW) results were also presented from application of the Pearson method and a semi-deterministic method (which are not presented in the current document). It was shown that results from all the methods applied were in good agreement with the experimental data. In [54] the semi-deterministic method was suggested as having a large potential to be used during design applications, due to its remarkable simplicity, comparable speed of calculations to that of pure deterministic and good agreement of the results with the more sophisticated methods applied.

The number of terms in the EDW application was also examined in [54], where it was shown that the improvement achieved using two instead of one term, that is, correcting the assumption that the failure function follows the Normal distribution is not important for the cases studied. Additionally, the improvement attained using also the fourth moment of the failure function is clearly smaller than the improvement attained using two instead of one term. However, there are differences, particularly in the estimations at the small values of probability of failure, which, nevertheless, are of the most interest, since during the structural design it is necessary to achieve a significantly small probability of failure. In Figure 5 the effect of the number of terms in the Edgeworth expansion is shown for two case of off-axis Gl/Ep layers, namely 15° and 20°, in comparison with experimental data, denoted “Exp.”, derived from [41] and application using the TH failure criterion as described in [54]. Results from Monte Carlo simulation (MC) are also shown for comparison purposes.

Experimental results from the OPTIMAT BLADES project (ENK6-CT2001-00552) [70] were used for an assessment of the presented reliability estimation methods. The results for the 5 basic material strength properties, i.e. the strength in tension and compression along the fiber direction and transversely to it, X_T, X_C, Y_T and Y_C, respectively and the in-plane shear strength are presented in [71]. The properties are assumed to follow the Normal distribution, although other distributions could be used as well for the modeling of the strength properties, e.g. the Weibull or the Log-Normal distribution. The statistical parameters of the strength properties, as obtained in [71] are shown in Table 1.

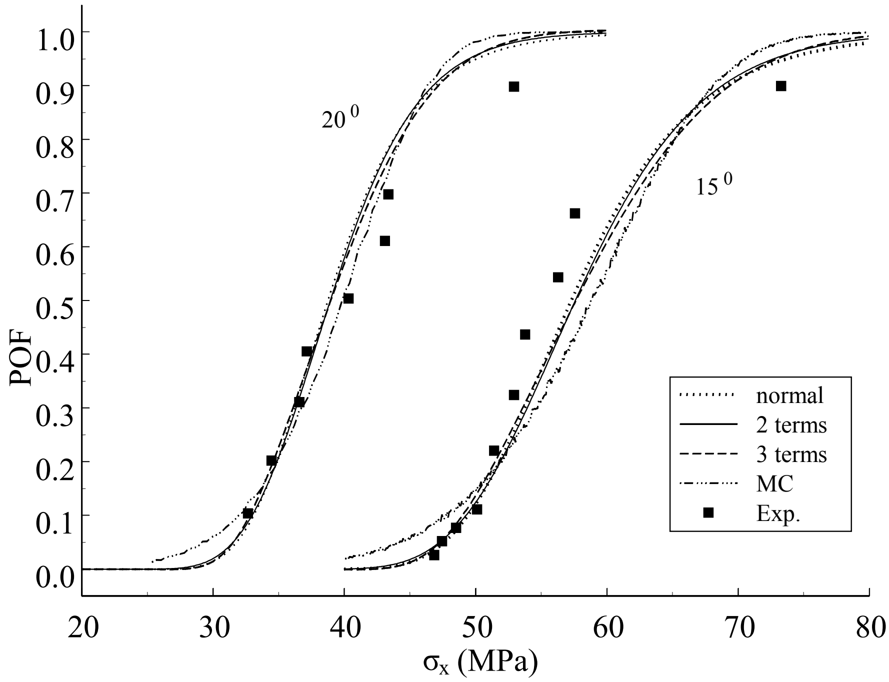

Figure 5 Effect of number of terms in the EDW expansion

Table 1: *Statistical parameters of the strength properties of UD layer*

Property	Mean Value (MPa)	St. Deviation (MPa)	C.O.V. (%)	Distribution
X_T	776.5	36.1	4.65	Normal
X_C	521.8	16.5	3.16	Normal
Y_T	54.0	2.6	4.81	Normal
Y_C	165.0	4.8	2.90	Normal
S	56.1	1.1	1.96	Normal

It should be noted that the experimental data, statistically analyzed and presented in Table 1 were obtained by tests conducted on coupons of 1 or 2 plates (for each parameter) only at University of Patras (while data reported in [70] are the results of tests conducted at more laboratories).

Figure 6 presents a comparison of the failure probability estimate as obtained by application of the Edgeworth Expansion method (EDW), the Monte Carlo simulation (MC) and the FORM method, for a unidirectional 10° off-axis layer under tension using the TH failure criterion, with experimental results obtained within the frame of the OPTIMAT BLADES project [70]. For the modeling of the material strength the data presented in Table 1 were used. It is clearly seen that the failure probability estimation is far from the experimental data, although the results of the three reliability estimation methods applied are in good agreement. Since similar deviations from the experimental results were obtained also for the 60° off-axis layer under tension, as well as the 10° off-axis layer under compression, the effectiveness of the failure criterion used, as well as the sensitivity of the prediction on the statistical data used for the basic variables of the problem were investigated.

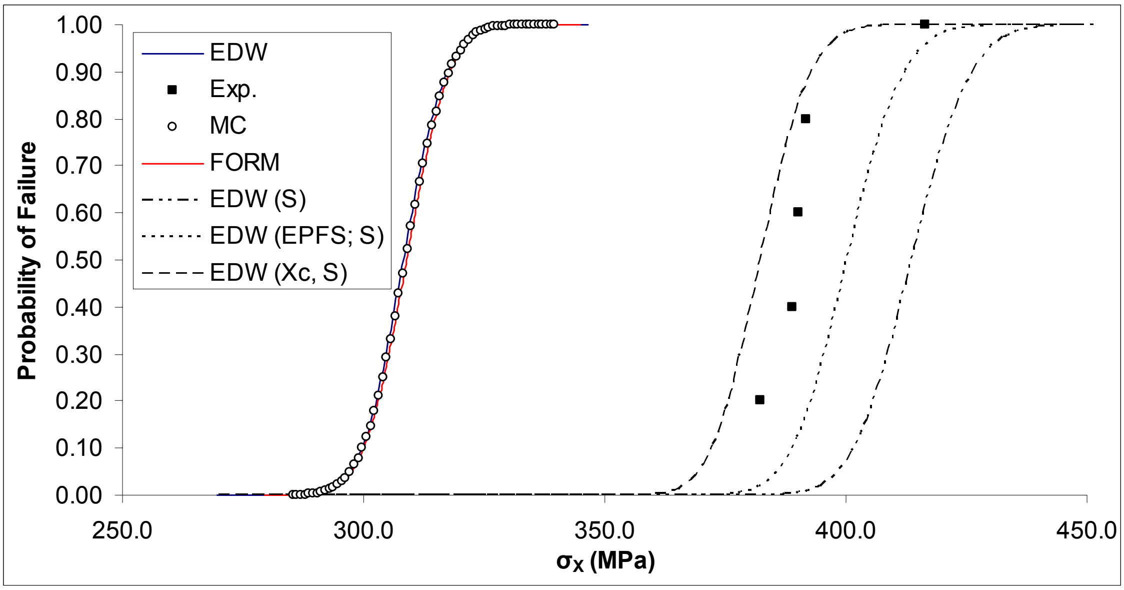

Figure 6 Failure probability estimation for an 10° off-axis Gl/Ep layer under uniaxial tension

In Figure 7 the purely deterministic results are presented for failure uniaxial stress estimation of off-axis loaded Gl/Ep layers in comparison with experimental data obtained from [70]. For the theoretical predictions, shown by the dashed line, the mean values of the strength material properties were used, while the failure criterion applied was the EPFS. The experimental data conducted by University of Patras (UP) in tension and compression for coupons with off-axis angles 10° and 60° are shown with square marks, while those obtained in tension by 30° off-axis coupons by Vrjie University of Brussels (VUB) and Risoe are marked with crosses and triangles, respectively. It is clearly seen that the theoretical predictions are not in good agreement with the test data. However, the theoretical predictions are much improved, if instead of the shear strength value obtained by tests conducted on ±45° coupons the respective value obtained by employing the experimental data of V-notched specimens is used [72], shown by the continuous line. The value for the shear strength obtained by the V-notched specimens is about 45% higher than that obtained by the ±45° coupons (81MPa versus 56MPa, respectively). At this point it should be noted that theoretical predictions obtained by employing the TH criterion are almost identical to results using the EPFS criterion. Moreover, it was shown in [73] that use of the mean values for the strength properties as presented in Table 1 in combination with Puck's failure criterion can effectively capture the failure stresses of the off-axis coupons. Nevertheless, the evaluation of the effectiveness of the various failure criteria falls not within the scope of the current work, see also section 2 and is only discussed wherever necessary.

Getting back to Figure 6 the failure probability estimation using the EDW method and applying the TH failure criterion, by assuming that the mean value of the shear strength S is that obtained from the V-notched specimens, denoted by "EDW (S)" is closer to the experimental results, but overestimating the strength obtained for the 10° off-axis coupons under tension. A small improvement is made by employing the EPFS criterion, denoted "EDW (EPFS; S)", but still the prediction is overestimating the reliability of the coupons under the specific test conditions.

A further improvement is obtained by replacing the mean value of the compressive strength in the fiber direction, X_C, obtained by experiments conducted using the OB test specimen geometry [70], by the respective strength using the geometry recommended in the ISO 14126 [74]. Tests conducted within the frame of the OPTIMAT BLADES project on the same material using the two different geometries resulted in two different (by 24%) estimations of the corresponding material strength property [71], namely a value of 686.3 MPa where obtained by use of the ISO geometry in contrast to the 521.8 MPa obtained using the OB geometry. Reliability predictions using as the mean value for the X_C the value obtained through tests on specimens having the ISO geometry are shown in Figure 6 by the long-dashed line, denoted "EDW (Xc, S)".

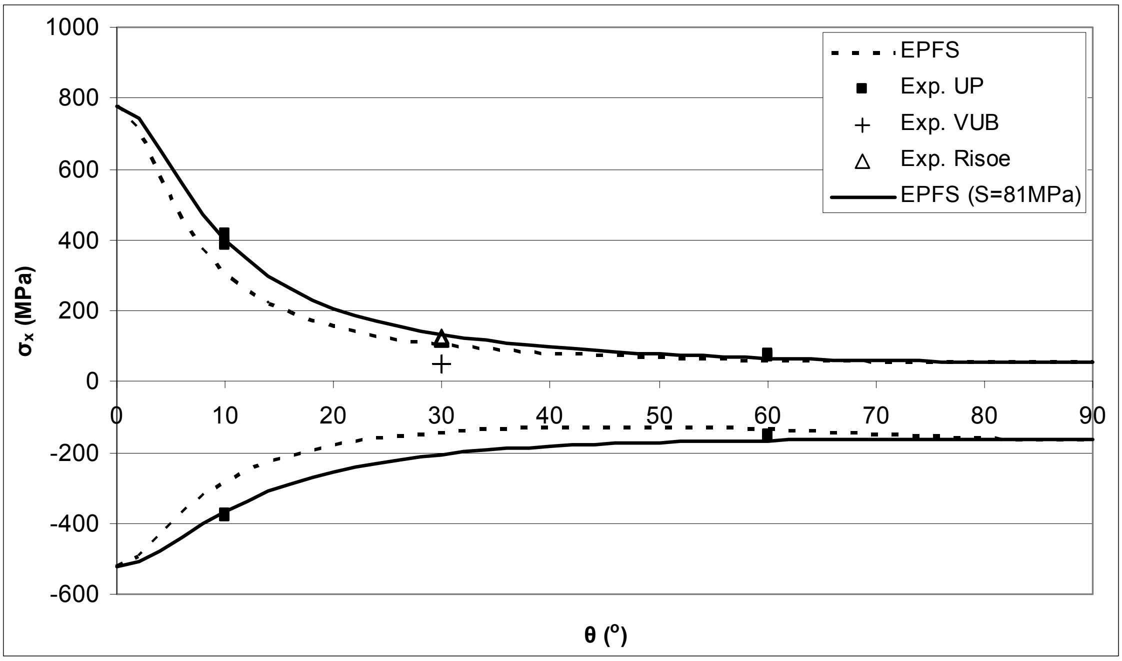

Figure 7 Deterministic Failure stress prediction with respect to the off-axis angle of unidirectional loaded Gl/Ep layers

In Figure 8 respective results are shown for the failure probability prediction of the 60° off-axis tension loaded unidirectional coupons in comparison to the experimental results obtained in [70]. Again the good agreement between results obtained with the EDW method and those from the MC simulation is seen, while both methods as applied result in large differences from the experimental data. However, for this case the corresponding modification applied on the theoretical estimations for the 10° off-axis layer and presented above resulted in moderate improvements for the failure probability prediction.

In Figure 9 the corresponding results are shown for the failure probability of the 10° off-axis coupons loaded in compression in comparison to the experimental results of [70]. For this case only the EDW estimation is shown with a continuous line, using the TH criterion and the statistical parameters of the strength properties as presented in Table 1. The results of the EDW method application using a mean value of the shear strength of 81 MPa instead of that shown on the above table is denoted in Figure 9 as "EDW (S)", which is in good agreement with the experimental data.

It should be noted, however, that the experimental results, as presented in Figure 6, Figure 8 and Figure 9 are too few for an probabilistic assessment on the one hand and on the other during the OPTIMAT BLADES project various subjects concerning the statistical analysis of experimental data and the parameters of the tests were revealed, which when trying to assess the effectiveness of probabilistic methods could be misleading. To be more specific, in Figure 10 the cumulative distribution of the strength along the fiber direction, X_T, as obtained by the experimental results of all static tension tests on UD coupons of the OB geometry and of the same Gl/Ep material (GEV206-R0300) reported in the OPTIMAT database are collectively shown by open circles (denoted "Exp."). These include results from various plates conducted by various laboratories within the frame of the project. In the same figure also the cumulative distribution of the experimental data used for deriving the statistical parameters of the material strength property, presented in Table 1, are also presented by squares, denoted "Exp. UP". As already mentioned, these were obtained by testing one or two plates at one laboratory (UP). Clearly the two experimentally derived cumulative distributions have differences. The difference in the mean value is about 2%. The coefficient of variation for all experiments (90 in total) is 6%, while for the 30 coupons tested by UP the coefficient of variation is 4.6%, yet the two cumulative distributions seem different. At the end of the project UP had 10 more coupons added in the database, thus, the complete set of experimental results for the X_T conducted by UP is shown in the same figure by crosses, denoted "Exp. UP Final". Comparing the two datasets, the differences are even smaller, that is, the mean values are different by less than 1% and the coefficient of variation of the final UP data set is 4.9%. Again the two experimentally derived cumulative distributions look different. To complete the discussion, the theoretical predictions using the Edgeworth expansion technique in combination with the TH failure criterion and the statistical parameters presented in Table 1, are also shown, denoted by "EDW". If this prediction is compared with the complete dataset containing results from all laboratories the difference is similar to the ones obtained for the other off-axis angles. Even if the prediction was compared with the original data set, i.e. the "Exp. UP" the comparison would not do justice to the theoretical results. This is due to the fact, that the strength was modeled as a Normal variate. The Normal cumulative distribution using the statistical parameters of Table 1 is shown with the red line, which is clearly in good agreement with the results obtained by applying the EDW method. A better model to the original dataset could perhaps be obtained if the Weibull distribution was used instead, shown in blue line. But still, more experimental data are necessary to obtain a valid model for the material strength properties and then compare with experimental results for more general stress conditions. In DOT/FAA/AR-00/47 [75] a rigorous analysis on the sampling method that should be followed as well as the statistical analysis procedure of the experimental data for the evaluation of composite material properties, taken as granted, however, that the properties follow Normal distribution.

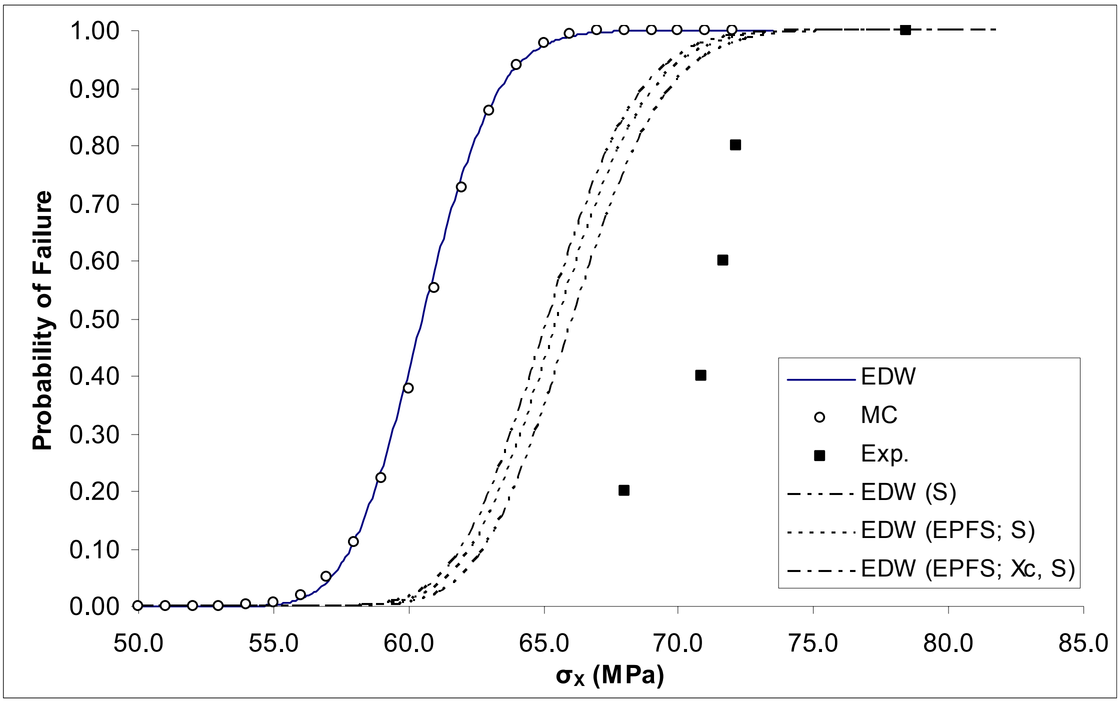

Figure 8 Failure probability estimation for a 60° off-axis Gl/Ep layer under uniaxial tension

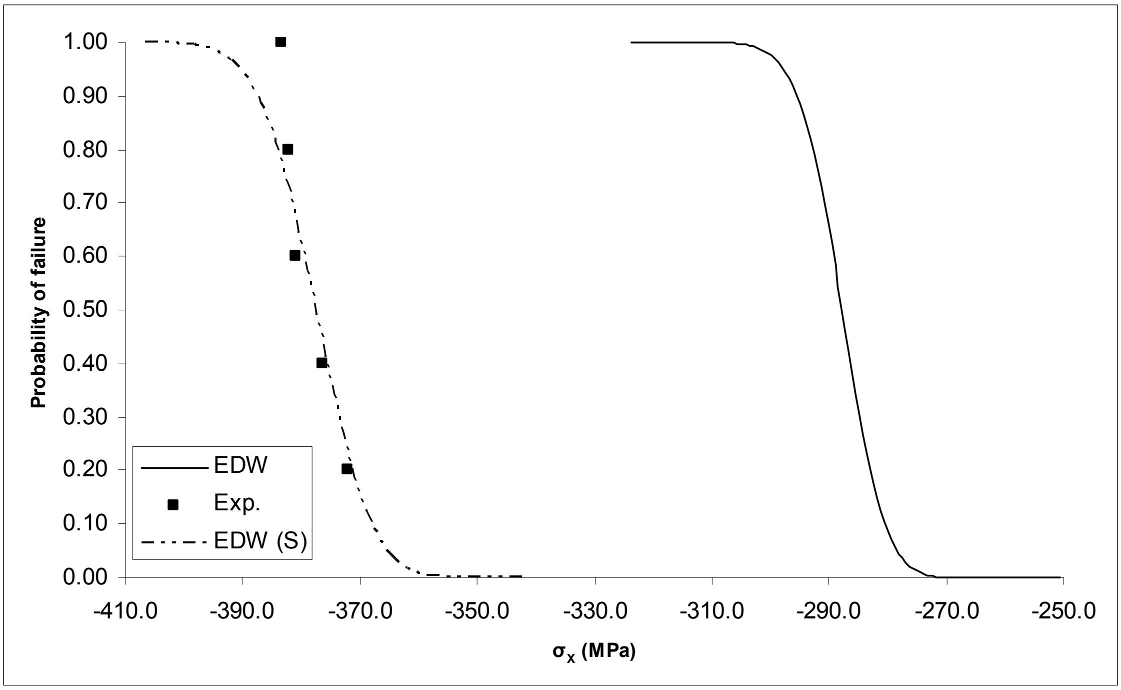

Figure 9 Failure probability estimation for a 10° off-axis Gl/Ep layer under uniaxial compression

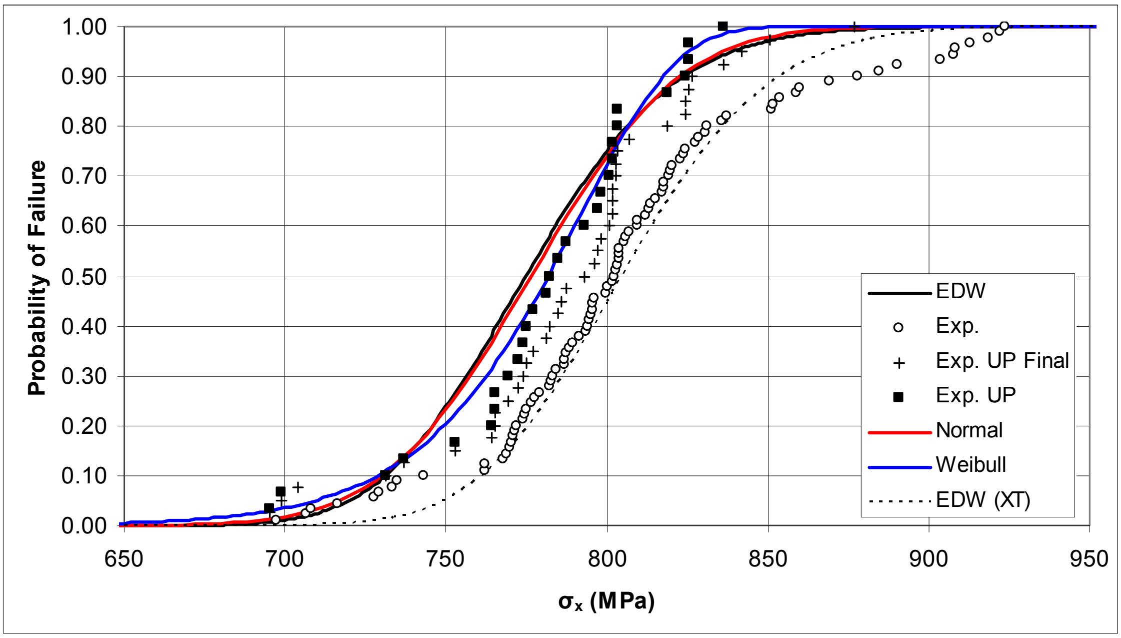

Figure 10 Cumulative distribution of the tensile strength along the fiber direction

As a final note, in Figure 11 the results on the strength in tension along the fiber direction from [70] are presented with respect to the plate number from which the coupons were cut. It is clearly seen that the results from some of the plates (e.g. around plate number 110) are higher than the rest, which could not be attributed to a specific coupon manufacturing or test parameter.

Due to the reasons explained in the above, as well as the fact that there are only few experimental results available for cases of general static in-plane loading, in this study the comparison with experimental data will be kept to a minimum.

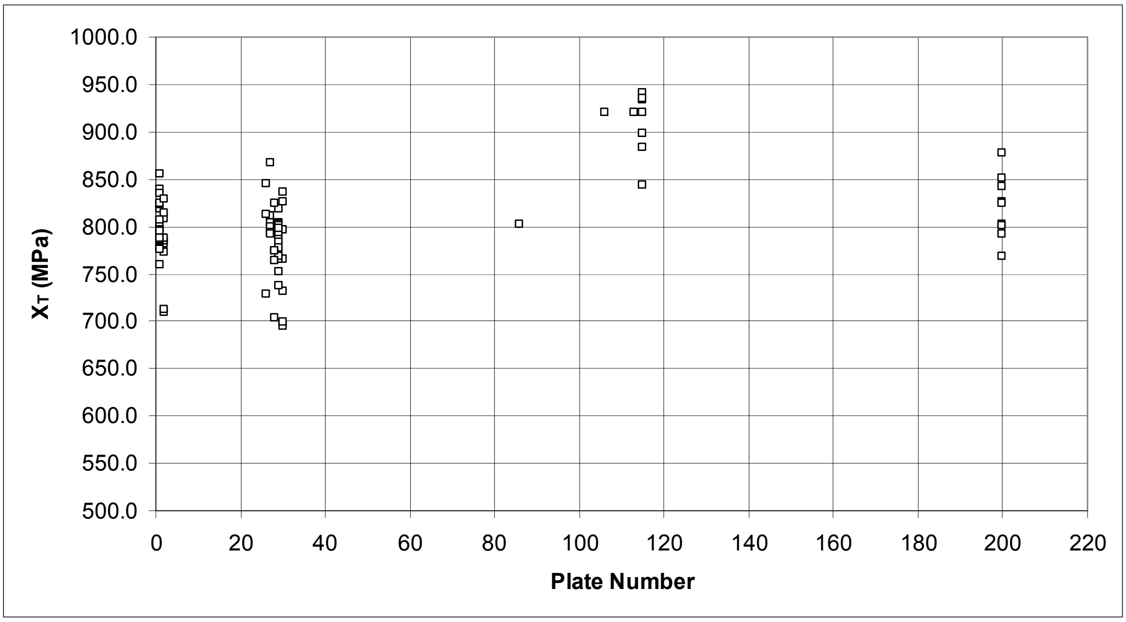

Figure 11 X_T strength with respect to the plate number

4.2 Comparison of reliability estimation methods in small probability of failures

In [61] the FORM method was presented for the estimation of the layer reliability under general in-plane stress condition. Similar to that work, conducted for Carbon/Epoxy material in this section the corresponding estimation is presented, for the Gl/Ep material. To this end, an off-axis layer is considered under axial and in-plane shear stress, σ_X and σ_S, respectively, while it is assumed that the strength properties are Normal variates with statistical parameters shown in Table 1. The applied axial and in-plane shear stresses, σ_X and σ_S, respectively are assumed deterministic and equal to 222 MPa and 185 MPa.

Figure 12 presents the reliability estimated through application of the FORM method for off-axis angles in the range of 22° to 37°. The off-axis angle is presented in the horizontal axis, while the vertical axis corresponds to the β index value. The secondary vertical axis shows an approximation of the corresponding reliability value. It should be noted that the β index value has only a meaning for the FORM method. For the other two reliability estimation methods, the results are shown in terms of the β index value, in order to facilitate comparisons in the whole range of reliability estimation (since the reliability scale is not linear). In the same figure results from the Monte Carlo simulation are also presented for comparison purposes. Results obtained through application of the EDW method are shown to underestimate the probability of failure for the cases of high reliability. Nevertheless, predictions are comparables for the off-axis angle range of medium probability of failure. Use of 3 terms in the EDW application does not affect the results, in the case of assuming that the basic variables follow the Normal distribution. Moreover, a second estimation of the reliability for the same problem is also shown (estimations in red, marked with “COV”), where instead of the coefficient of variation experimentally determined and shown in Table 1, the strength properties of the material are assumed to have twice as much standard deviation. In this case, although again the results of the EDW are overestimating the reliability of the layer, the difference is less, than the previous case.

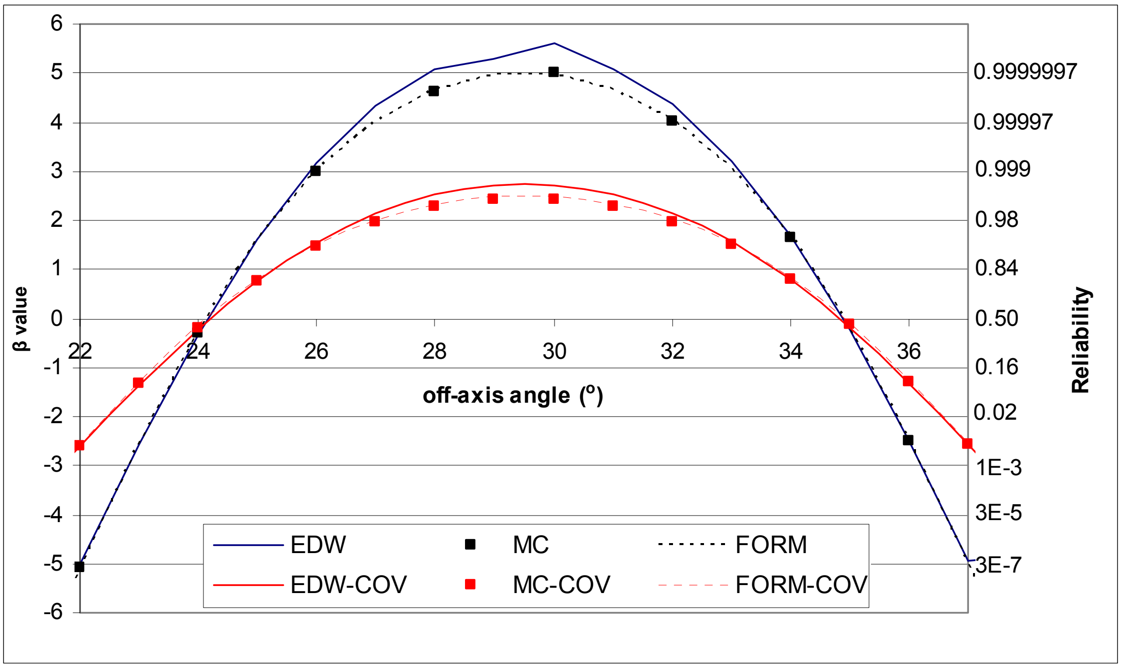

Figure 12 Comparison of reliability estimation from applied methods

4.3 Lamina failure locus

Several times during the deterministic design lamina failure loci for various off-axis angles are used for comparing the behavior of the layer under general in-plane loading conditions. In order to attain results comparable with the reliability level achieved using partial safety factors during a deterministic design, the failure prediction has been conducted for specific reliability levels.

The failure prediction for a specific reliability level for the EDW, FORM and MC method is performed by searching in the space of the in-plane stresses, the combination that gives the required level of reliability. On the contrary, the drawing of a failure locus under a given reliability level through application of the partial safety factors is simply conducted by replacing the mean values of the strengths with the design values and solving for the applied stress.

For example, comparison of the failure locus predictions with the various methods in shown in Figure 13 for an on-axis layer under combined axial and shear stress at a reliability level of 0.9999, i.e. $P_F = 10^{-4}$. The material strength properties for the layer are assumed to be that given in Table 1, while the failure criterion used is the EPFS criterion, expressed by Eq.3 with the off-diagonal term given by Eq. 7. The overall agreement of the EDW prediction, shown by the continuous line, with that of the FORM, shown by the dash-double dotted line, and the MC, shown with square marks, which is on top of the results of the FORM method, is relative good. This is strengthened by the fact that the EDW method has no convergence problems (as the FORM method) and no iterations (as in both the FORM and the MC method), which constitutes the EDW an attractive method for use during the design phase of a composite material wind turbine blade.

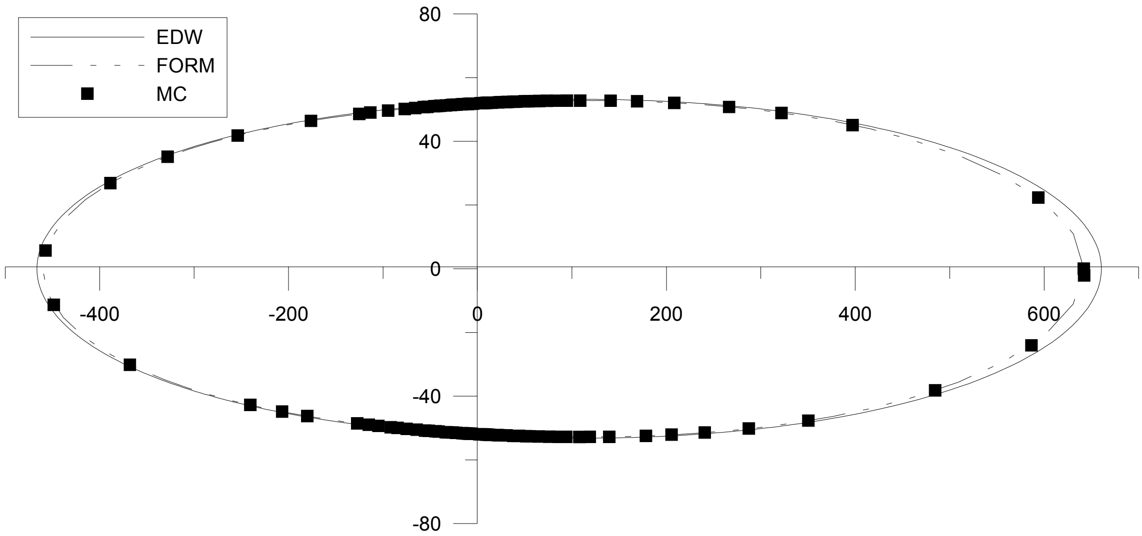

Figure 13 Failure locus for on-axis layer of Gl/Ep, comparison of various methods

4.4 Lamina failure locus on the strain space

Analogous results are obtained if instead of the stress space, the failure locus is estimated on the strain space. Starting by assuming a linear relationship between stress and strain on the natural system of the layer:

$$\sigma_i = Q_{ij}\varepsilon_j \qquad i, j = 1,2,6 \qquad \text{Eq. 36}$$

where Q the in-plane stiffness matrix, which is given for an on-axis layer in terms of the elastic properties of the orthotropic lamina (the elasticity modulus in the fiber direction and transversely to in, E_1 and E_2, respectively, in-plane shear modulus, G_{12} and the Poisson ratio, v_{12} by [76]:

$$[Q] = \begin{bmatrix} \dfrac{E_1}{1-\dfrac{E_2 \nu_{12}^2}{E_1}} & \dfrac{\nu_{12} E_2}{1-\dfrac{E_2 \nu_{12}^2}{E_1}} & 0 \\ \dfrac{\nu_{12} E_2}{1-\dfrac{E_2 \nu_{12}^2}{E_1}} & \dfrac{E_2}{1-\dfrac{E_2 \nu_{12}^2}{E_1}} & 0 \\ 0 & 0 & G_{12} \end{bmatrix} \qquad \text{Eq. 37}$$

the tensorial failure polynomial of Eq. 3 can be expressed in terms of the strains developed on the layer [76]:

$$H_{ij}\left[Q_{ik}\varepsilon_k\right]\left[Q_{jl}\varepsilon_l\right] + H_i\left[Q_{ij}\varepsilon_j\right] - 1 \le 0 \qquad i, j, k, l = 1,2,6 \qquad \text{Eq. 38}$$

where summation of terms with repeated indices is assumed.

Assuming that the elasticity properties of the layer are deterministic and employing linear elasticity, the failure locus of an on-axis layer on a specific reliability level is obtained by applying exactly the same procedure as for the failure locus on the stress space, namely, using the search algorithms for the strain combination that results to the necessary reliability.

For example on Figure 14 the failure locus for an 45° off-axis layer on the $\varepsilon_x - \varepsilon_y$ plane for reliability level 0.9999, i.e. $P_F=10^{-4}$ is shown by use of the EDW and the FORM method with lines in black color. The material strength properties are shown in Table 1, while the elastic properties of the material were taken as the mean values of the experimental results presented in [71], that is E_1 = 39.04 GPa, E_2 = 14.08 GPa, v_{12} =0.291 and G_{12} = 4.24 GPa. On the same graph the failure locus prediction of the same case but assuming that the standard deviation of the basic variables, that is the strength properties is shown for the two reliability estimation methods with lines in red (EDW-VAR, FORM-VAR). Clearly, the results of the two methods using the original variance of the strength properties are in good agreement, while the results of the EDW prediction are non-conservative in comparison to the FORM method results when using higher variance for the properties.

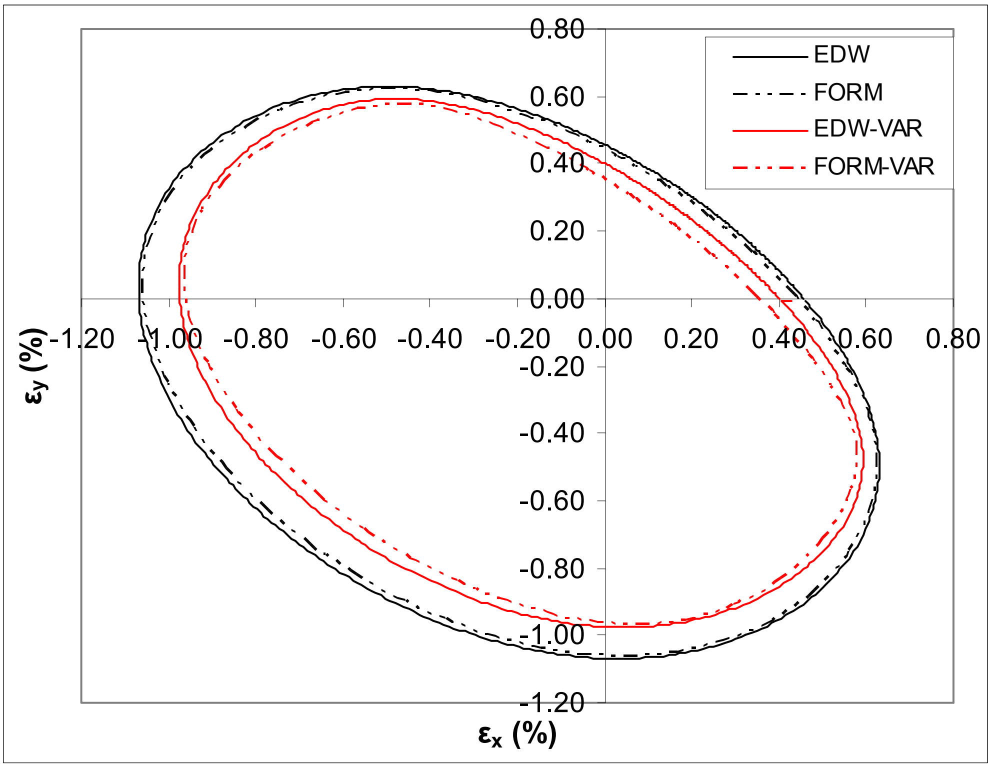

Figure 14 Comparison of failure locus on strain space

4.4.1 Considering the stochastic nature of the elastic material properties

In case the elastic material properties are also assumed as stochastic parameters of the problem, then with application of the reliability estimation methods presented in the previous sections of the current document, the failure probability of the layer can be estimated in any combination of axial and shear strains (assuming an in-plane stress state).

The application of the Monte Carlo method is similar to the one presented, where only the strength properties were assumed stochastic, by generating random variables, namely the five strength and the four elasticity properties of the layer and solving the deterministic problem in every iteration, while for each iteration it is recorded whether a safe or a failure condition has been obtained.

For the application of the EDW and FORM method, the partial derivatives of the failure function with respect to the elastic properties of the layer, which are incorporated in the function through the applied stresses, need to be estimated. Specifically, for the application of the EDW method derivatives of the failure function up to the second order with respect to the basic variables, namely the strength and elasticity properties, are needed for a better estimation of the moments of the failure function. For the application of the FORM only the first order derivatives are necessary. To this end, starting with the failure function and with continuous back substitutions using the stress-strain relationships and the equations of the in-plane stiffness matrix with respect to the 4 elastic properties characterising the composite material layer, the abovementioned reliability estimation methods can be applied for the failure probability prediction of the layer given the strains developed.

Failure loci at specific reliability levels in the strain space where presented in [66] considering as stochastic parameters apart from the failure stresses and the elasticity properties, also the geometric parameters of the layer (the thickness, the orientation angle, etc.), while the method applied was FORM. In the examples presented in this section the geometric parameters of the layer are assumed deterministic.

In Figure 15 the failure locus of an 45° off-axis layer is presented on the $\varepsilon_X - \varepsilon_y$ plane for a reliability level 0.9999, that is, $P_F = 10^{-4}$ as obtained by the application of the EDW method and the FORM. In the same graph the corresponding prediction, taking into consideration also the variability of the elastic properties (failure loci denoted by “elast”) are shown. For the case where the elasticity properties are also assumed to be stochastic the statistical parameters employed are presented in Table 2, where data where taken from [71].

Table 2: *Statistical parameters of the elastic properties of UD layer*

Property	Mean Value	St. Deviation	C.O.V. (%)	Distribution
E_1 (GPa)	39.04	1.32	2.64	Normal
E_2 (GPa)	14.08	0.33	2.31	Normal
v_{12} (-)	0.291	0.027	9.28	Normal
G_{12} (GPa)	4.24	0.10	2.34	Normal

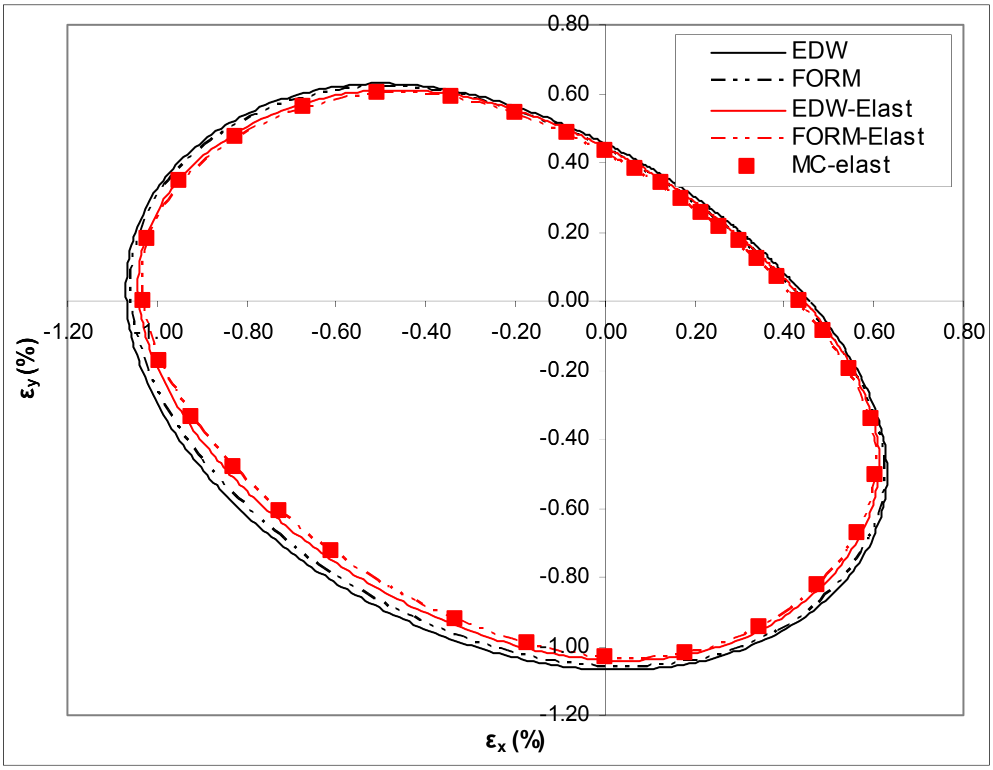

Figure 15 Comparison of Failure loci on the strain space with and without considering the elastic properties as deterministic

5. Laminate failure probability

A laminate can be modelled as a system of many components, each one of them characterized by its own failure function. The failure of the laminate may in turn be characterized either by the failure of the first ply (FPF) or by the total failure, that is the successive failure of all layers in the laminate up to the failure of the last ply (LPF). When the laminate is designed so that on the limit state the FPF condition is not exceeded, the aim is to avoid any development of micro-cracks in the material. On the other hand, when the design is performed in such a way, so as to not exceed the LPF condition on the limit state, then the scope is to exploit in the best possible way the material and therefore, optimize the use of the laminate.

In the current work, the failure of the laminate is considered to be characterized by the FPF method for two reasons. The first is that for the LPF prediction assumptions for the material degradation factors are needed during the estimation of the failure load. However, since there a lot of discussions concerning the adequate degradation methodology including the material degradation factors, as for example in [76] and [77], while the most usual way of determining these degradation methodology it to verify the assumptions with experimental results, the choice of a methodology would incorporate larger uncertainty in the reliability estimation, this way masking the results of the current work. The second reason is that although the FPF assumption underestimates the final failure of the laminate, this assumption constitutes a conservative albeit safe approach during the design of composite material structures as wind turbine blades.

In order to pass from the estimation of the failure probability of the lamina to that of the laminate, that is, to the basic element of the structure, in a probabilistic approach the laminate should be considered as a system. Therefore, considering the laminate of n plies as a system, where the elements of the system are the n layers, according to the FPF hypothesis, an adequate model of the system would be the series system. On the contrary, in the case of the LPF, where the failure of the system is attained by the failure of all the elements, then the assumption of a parallel system is more appropriate. Since the design is considered to be performed based on the FPF methodology, the analysis of a series system will result to the probability of failure of the laminate.

Consider a laminate of k layers. Each layer will have different limit state functions. Let the limit state function of the i-th layer be expressed by:

$$G_i(X) = G_i(x_{1j}, x_{2j}, \ldots, x_{nj}) \qquad j = 1, 2, \ldots, k$$

for which the event of failure is:

$$A_i = [G_i(X) \le 0]$$

Then, the complementary event of A_i will be the safe event, that is:

$$\overline{A_i} = [G_i(X) > 0]$$

The safe condition of the laminate, in the case of the FPF assumption, is described by the event that "<u>none of the k layers is in a failure state</u>", that is:

$$\overline{A} = \overline{A_1} \cap \overline{A_2} \cap \ldots \cap \overline{A_k}$$

The event of failure of the laminate (the system) will be described by:

$$A = A_1 \cup A_2 \cup \ldots \cup A_k$$

The above equation implies that one or more layers are in a failure condition. Theoretically, it follows from the above, that the probability the system to be in a safe state is expressed by the spatial integral:

$$P_S = \int_{\left(\overline{A_1} \cap \overline{A_2} \cap \ldots \cap \overline{A_k}\right)} \int \ldots \int f_{X_1, X_2, \ldots X_k}\left(X_1, X_2, \ldots, X_k\right) dX_1 \, dX_2 \ldots dX_k$$

The calculation of this probability or the respective probability of failure of a laminate through the above spatial integral is in general difficult. In most of the cases, approximate solutions are necessary. From this point of view, it would be also useful to have the respective probability limits. The limits for the probability of non failure for the laminate, where it is assumed that the failures of the layers are positively correlated are given by [48]:

$$\prod_{i=1}^{k} P_{s_i} \le P_S \le \min_i P_{s_i}$$

where the reliability of the i-th layer is denoted as P_{s_i}. Similarly, the respective limits for the failure probability of the laminate are given by [48]:

$$\max_i P_{f_i} \le P_F \le 1 - \prod_{i=1}^{k}\left(1 - P_{f_i}\right)$$

The range between the upper and the lower limit is obviously dependent by the number of layers and the relative values of their probabilities. For example, if there is a dominant layer failure then the reliability of the system will depend on the probability of failure of this layer and in some cases it would be appropriate to estimate the probability of failure of the laminate only by the probability of failure of the dominant layer. In this case the limits range will be narrow. In general, however, the limits will be rather wide, especially if the number of independent failure modes is large. It should be noted that in the case of composite materials, which are studied in the current work, the independent modes of failure are equal to the number of layers in the laminate, if each layer is characterized by its own failure function (as described in former sections of the current document).

Specifically, in the case that the failure of each layer is completely independent of the failure of any other layer in the laminate, the probability of failure and the reliability of the laminate are given by following equations, respectively:

$$P_F = 1 - \prod_{i=1}^{k}\left(1 - P_{f_i}\right) \qquad \text{Eq. 39}$$

$$P_S = \prod_{i=1}^{k} P_{s_i} \qquad \text{Eq. 40}$$

In case the failures of the layers are positively correlated the probability of failure and the reliability of the laminate are given by following equations, respectively:

$$P_F = \max_i P_{f_i} \qquad \text{Eq. 41}$$

$$P_S = \min_i P_{s_i} \qquad \text{Eq. 42}$$

In the case where a more narrow estimation of the failure probability limits for the laminate is sought, the correlation coefficient between the failures of the layers should be determined. However, the correlation coefficient, alike the degradation factors, is difficult to be estimated, since the degree of correlation of the layer properties in a laminate depends for example by the manufacturing procedure of the laminate, the spatial distribution of the fabric properties and the lamination sequence. In particular, for similar layers it can be assumed that the more standardized manufacturing procedure the more the positive correlation of the layer properties.

For example, during experiments conducted for the determination of the strength properties of a composite material, coupons are manufactured by plates of several layers, depending on the thickness of a layer, which are subjected to a uniaxial stress condition up to the final separation of the specimen, or the condition where the coupon is unable to support larger load. Nevertheless, in this case, while we have a laminate, we assume that the properties of this laminate coincide with the properties of a single layer. In other words, it is assumed that the properties of all layers in the laminate are equal. However, behind this hypothesis, the assumption that the properties of the layers are perfectly positively correlated is hidden.

In [78] and [15] the conjecture of the laminate as a series system or as a parallel system, estimating the upper and lower limit of the failure probability depending on each conjecture, assuming however, that the failure of each layer is an independent event of the failure of another layer is presented. In [79] it is suggested that the laminate should be thought of as a series system, where the laminate failure coincides with the failure of the first ply. For the reliability estimation the upper and lower probability limits calculation is proposed in this work, as for example the use of the Ditlevsen limits. A similar approach is also proposed in [80]. In the same article, nevertheless, it is noted that for the specific problem at hand, where $[\pm\varphi]_n$ laminates are studied, the probability of failure of each layer are equal, therefore, the probability of failure of the system (i.e. the laminate) is equal to the probability of failure of any layer. On the other hand, in [81], while it is recognized that the failure of the laminate is not always coinciding with the failure of the first ply it is assumed that as a first approximation of the problem the conjecture of the laminate as a series system is essential, since the probability estimation of the last ply failure presumes a progressive failure analysis. Moreover, it is assumed that all layers are correlated, so that $P_F = \max_i P_{f_i}$, since the hypothesis that the layers are uncorrelated means that the correlation coefficients are known.

In [82] the last ply failure is estimated. For this estimation, if the failure of a layer is noted, during the load increase, then the reason of the failure of the layer is investigated to find the particular mode of failure (shear, matrix failure, etc.) and depending on that result, the elastic properties correlated with that failure are degraded to negative values. During a further load increase (which for the specific case is an internal pressure), the elastic properties of the failed layers are continued to be taken as negative until the corresponding stresses return to zero. After that, the elastic properties (and the corresponding stresses) are assumed to be zero. The failure of the laminate is assumed to be when a failure of a layer due to fiber failure is noted or when the stiffness matrix of the laminate gets negative.

A very good description of the modes of final failure of a laminated plate is given in [62]. It is noted that for estimating the probability of the laminate to be in a safe state with more accurately, an investigation of each sequence of ply failures up to the final failure of the laminate should be conducted and then, the failure probability of the laminate should be approached by employing theory of probability of events. After the first ply failure, for the estimation of the next ply failure, it is assumed that the elastic properties of the layer that failed are equal to zero. Due to the size of the problem for the estimation of all failure sequences and the probability of their occurrence, even in the case that the laminate comprises 10 layers, only the most probable of all the possible failure sequences are selected for the estimation of the required probability of the system. Moreover, it is mentioned that in the case of composite materials it is rare in practice to have a laminate that can sustain a load increase after the third layer has failed, while it is usual that the laminates can sustain loads even after the failure of the first and second ply in the lamination sequence. For the approximation of the final failure probability of the laminate the procedure of load increase should be also investigated. In case the external load is increased monotonically, then the solution is concentrated only on the investigation of the failure sequence of the layers. In a different case, that is, when the externally applied load can be increased with different ways, then an investigation on the load increase procedure should be also undertaken.

5.1 Laminate failure locus prediction at specified reliability level

For the estimation of the failure probability of a laminate under general in-plane stress state the classical lamination theory (CLT) is employed for the determination of the stress tensor developed in each laminate. The elastic properties of the material as well as the in-plane stress resultants of the laminate in the first phase are assumed to be deterministic. Therefore, the determination of the stresses developed in each layer does not entail any difficulties.

In case the elastic properties of the orthotropic material (layer), namely the modulus of elasticity in the fibre direction and transversely to it, as well as the in-plane shear modulus and the Poisson ratio, are assumed to be stochastic, the stresses developed in each layer due to the externally applied loads also depend on the variability of the elastic properties. Thus, for the cases of analytical approximation of the failure probability (i.e. using FORM or EDW methods) the failure function is expressed not only in terms of the moments of the strength properties, but also in terms of the moments of the layer stresses, which in turn can be expressed in terms of the elastic properties of each layer.

Consequently, for the analytical methods used in this work, in case the material elastic properties are considered as basic variables, then the partial derivatives of the failure function with respect to these variables is required, analogous to the procedure used in section 4.4.1.

As an example, the FPF loci at a reliability level of 0.9999 ($P_F = 10^{-4}$) of a $[\pm 45/0_2]_S$ laminate, which is characteristic of that used in wind turbine rotor blades is shown in Figure 16. The material considered is the Gl/Ep used in the OPTIMAT BLADES project with strength properties as presented in [71] and shown in Table 1. For the statistical modelling of the material elastic properties, the data from [71] have been used too, as shown in Table 2. The failure function applied is the Tsai-Hahn criterion, given by Eq. 3 with the off-diagonal term given by Eq. 6.

In this figure the Edgeworth estimation is shown with a continuous line when only the strength properties are assumed to be stochastic, denoted by EDW-S, while with the dotted line (EDW-E), the respective results are shown when both strength and stiffness are assumed to be stochastic. Simulation results using the Monte Carlo and the FORM method are also presented. In particular MC results considering only the strength properties as stochastic (MC-S) are presented with squares, while the respective results using the FORM method are denoted as FORM-S. When the stochastic nature of the elastic properties is also taken into account results using the FORM method are denoted FORM-E. One should first notice the small difference between the analytical and the MC results, noting that the EDW estimation is overrating by a little the structural load carrying capacity of the laminated plate. The results using the FORM method, when considering only the strength properties as stochastic variables are identical with the respective Monte Carlo data. Also in the cases of considering the elastic material properties as stochastic too, the results of the FORM and the MC method are in very good agreement, although they are not shown here for clarity. The next most important observation is the effect of the elastic properties variability on the failure locus prediction. For both FORM and EDW results the stochastic nature of the material elastic properties is affecting the failure locus.

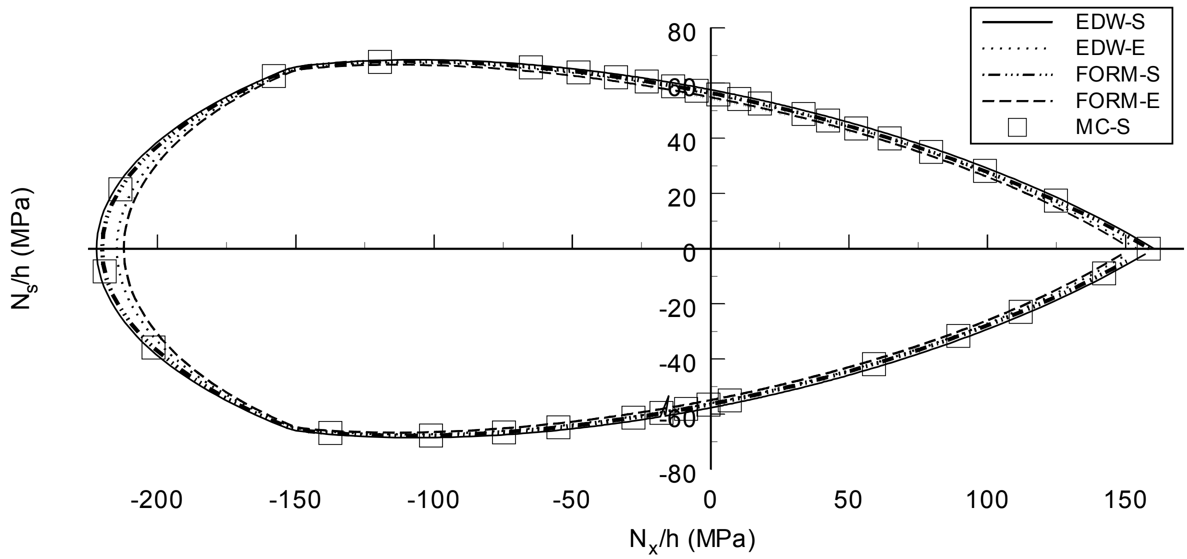

Figure 16 FPF failure locus of a Gl/Ep $[\pm 45/0_2]_S$ laminate at $P_F=10^{-4}$

5.2 Comparison with deterministic design

While in the IEC 61400-1 [39] there is no clear reference on the target probability of failure of the rotor blade, it is assumed in accordance with similar structures that the design is conducted for a probability of failure of 10^{-4}. Therefore, it is possible to compare the deterministic and the probabilistic estimation of the failure locus. In Figure 17 for example, the failure locus of a $[0/90]_S$ laminate is presented when only the strength properties are considered as stochastic. For the estimation of the deterministic failure locus, the strength properties were derived from the experimental data by applying the general partial safety factors on the characteristic material properties of 95% survival probability with 95% confidence limit. It is clearly seen that the deterministic failure locus, denoted by IEC, underestimates the structural load carrying capacity of the laminated element on the 3rd quadrant. Again the results with application of the Edgeworth (EDW) method are in good agreement with that of the FORM method, which are closely following the MC data. When also the elastic material properties are considered as random the failure locus is located between the probabilistic and the deterministic results, however it is not shown here for clarity.

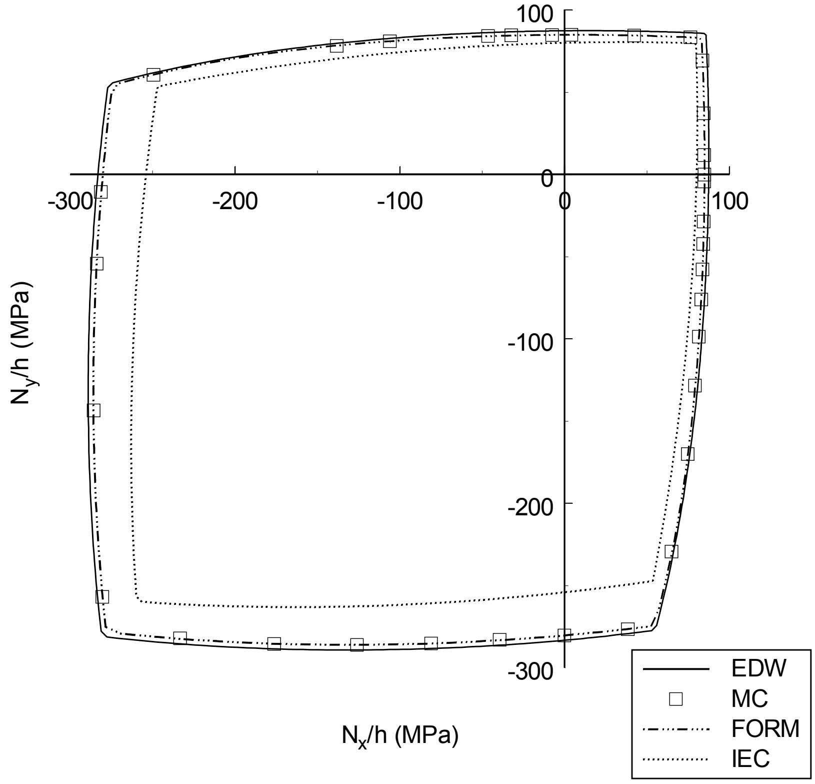

Figure 17 Comparison between probabilistic and deterministic failure loci for a $[0/90]_S$ Gl/Ep laminate. $P_F=10^{-4}$

Nevertheless, it should be noted, that the results presented herein depend on the amount of material property variability. For other test cases, where the strength and elasticity properties had a coefficient of variation between 10% and 20% the effect of the stiffness variability was much pronounced. Moreover, it was found that for this case the deterministic locus derived following IEC 61400-1 [39] was overestimating the ability of the structure to carry the load in the 3rd quadrant. The corroboration of EDW predictions by simulation results is satisfactory for the example presented, which is even strengthened by the fact that the numerical effort for the FORM estimation is much greater than that needed for the EDW prediction, while it is even larger when considering the MC simulation.

6. Conclusions

The work conducted for the adaptation and/or development of methods suitable for the probabilistic strength analysis of composite rotor blades was presented in the current report. Work performed included the development of numerical procedures for determining the strength of a composite laminate, using various failure criteria, by taking into account the stochastic nature of anisotropic (strength and stiffness) material properties.

An analytic approximation, namely the Edgeworth Expansion Technique, was presented for the estimation of the failure probability of a laminated composite plate under general in-plane loading, considering the material mechanical properties as being stochastic. Results were compared with the advanced first order second moment method and Monte Carlo simulation data and were found in good agreement for most of the cases. Model assessment and validation was also performed by comparing with the experimental results wherever possible, basically from tests on unidirectional reinforced Gl/Ep composites under uniaxial and bi-axial loading.

Results presented, focusing on the effect of the variability of the various material property groups, i.e. strength and elasticity, revealed that neglecting the stochastic nature of the material stiffness could result in an overestimation of the structural reliability. This however, depends on the amount of variation of the elastic material properties.

Finally, a direct comparison has been performed between deterministic and probabilistic design for laminates encountered in the rotor blade structures. It revealed that in general there are some differences in the prediction of failure for specified target reliability, especially when the laminate is under compression.

The gain in using the analytic method presented herein is in the minimization of computation time, since both MC simulation as well as the widely accepted FORM are rather time consuming. Additionally, the FORM method does not converge for all combinations of applied stresses and lamination sequences, which makes the use of the method during the initial phases of structural design of wind turbine rotor blades not attractive.

Within the UPWIND project it is planned to implement the numerical model in appropriate software routines in the form of pre- and post-processors that can be used along with current aeroelastic codes. This will lead in quantifying blade design reliability.

7. References

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