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Investigation of turbulence measurements with a continuous wave, conically scanning LiDAR

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Risø DTU
National Laboratory
for Sustainable Energy

Investigation of turbulence measurements with a continuous wave, conically scanning LiDAR

Rozenn Wagner, Torben Mikkelsen, Michael Courtney

Report number ex. Risø-R-1682(EN)

Risø National Laboratory for Sustainable Energy
Technical University of Denmark
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March 2009

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Department: Wind Energy Division

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Abstract (max. 2000 char.):

LIDAR systems are getting more and more accurate and reliable. It has been shown many times that the mean horizontal wind speed measured by a lidar over flat terrain compares very well with that measured by a cup anemometer. But can a lidar measure turbulence?

Here we investigate the case of a continuous wave, conically scanning Zephir lidar. First, the wind speed standard deviation measured by such a lidar gives on average 80% of the standard deviation measured by a cup anemometer. This difference is due to the spatial averaging inherently made by a cw conically scanning lidar. The spatial averaging is done in two steps: 1) the weighted averaging of the wind speed in the probe volume of the laser beam; 2) the averaging of the wind speeds occurring on the circular path described by the conically scanning lidar.

Therefore the standard deviation measured by a lidar resolves only the turbulence structures larger than a length scale depending on the circle diameter and the mean wind speed (range of magnitude: 100m). However, the Zephir lidar gives another turbulence quantity, the so-called turbulence parameter, which can resolve turbulence structures with a smaller length scale.

In this paper, we suggest a volumetric filtering of the turbulence to represent the effect of the spatial averaging operated by a lidar when measuring the wind speed. We then evaluate this model by comparing the theoretical results to experimental data obtained with several Zephir systems, for both turbulence quantities.

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Preface

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1 Introduction

LiDAR systems are more and more attractive for the wind energy industry, because of their ability to measure mean wind speed profiles up to about 200m. Their accuracy has increased significantly over the last 5 years. 10 minutes mean wind speed measured by some LiDAR systems compare now very well with cup anemometer measurements [1].

Meanwhile another quantity of great interest for the wind industry is the turbulence. The wind speed standard deviation measured by a remote sensing device is inherently different from the one measured by an anemometer. Indeed an anemometer measurement can be considered as a point measurement, whereas a LiDAR system measures over a volume. In this paper we consider the volume averaging effect obtained with a continuous wave (cw) QinetiQ ZephiR LiDAR.

Firstly, for such a system, the measurement at a given height is achieved by focusing the laser beam at the required distance. The wind speed component along the laser beam (the radial speed) results from a weighted average over the probe volume. Sjöholm et al. [2] proposed a volumetric filtering of the the turbulence to represent the effect of averaging within the probe volume for a ZephIR LiDAR. They have shown a good agreement between this model and experimental data.

Secondly, a ZephIR lidar device conically scans the air at a rate of one revolution per second. The horizontal wind speed results from the radial speeds obtained over one or three complete rotations. Therefore there is an averaging of the wind speed over a circular path centred above the scanning lidar. In this paper we describe the spatial averaging operated by a conically scanning lidar when measuring the wind speed and we focus on the effects of the conical scanning. We then refine the model suggested by Mikkelsen & Jørgensen [3] and evaluate these improvements by comparing the theoretical results to experimental data.

To realize such a comparison, we can look at two quantities given by a ZephIR LiDAR: the wind speed standard deviation and the "turbulence parameter". These two parameters give information on different scales of turbulence. Although the first one seems obvious, the standard deviation from a lidar is different from the corresponding standard deviation measured by a cup anemometer because of the spatial averaging. On the other hand the turbulence parameter is not that obvious and we further explain what this quantity represents in terms of turbulence covariances.

This investigation therefore has two main objectives:

- to define and evaluate a simple model for the filtering effect of the conical scanning;
- to provide an interpretation of the standard deviation and the turbulent parameter measured by a ZephIR lidar.

2 Volume Averaging: theory

2.1 Staring LiDAR _ Probe length averaging

The QinetiQ ZephIR lidar consists of a continuous wave (cw) coaxial laser and detector system [4]. In order to explain the way of operation of the system, we first consider a LiDAR staring in one direction. It emits a cw laser radiation, and detects the Doppler shift of the signal backscattered by the particles convected by the air. This Doppler shift is proportional to the radial wind speed, *Vr*, i.e. the projection of the wind speed vector, $\vec{u} = \{u, v, w\}$, in the laser beam direction [5].

A cw lidar measures the radial wind speed at a given distance *r* by focusing the laser beam at that distance. The power of the laser is then distributed along the beam with a distribution that has its maximum at the focus point. The radial wind speed *Vr* (*r*) measured by a cw lidar focused at a location *r* can generally be expressed by integration along the laser beam of the wind field $\vec{u}$ projected in the direction of the beam:

$$Vr(r) = \int_{-\infty}^{\infty} \varphi(s)\vec{n}.\vec{u}(s\vec{n} + r\vec{n})ds \qquad (1)$$

where $\vec{n}$ is a unity vector along the beam and $\varphi(s)$ is the spatial volume averaging function. For a focused cw coherent Doppler lidar, the spatial volume averaging function is given by the power distribution along the beam which can be approximated by a Lorentzian function [6]:

$$\varphi(s) = \frac{1}{\pi}\frac{z_R}{z_R^2 + (s-r)^2} \qquad (2)$$

where *s* is the distance from the focus point along the beam, *r* the focus distance or range and z_R is the so-called Rayleigh length, related to the depth of focus. The depth of focus or probe length is generally defined as $2z_R$. For the series ZephIR lidar, the Rayleigh length was estimated to be: $z_R = 0.0013r^2$ [1]. According to the Lorentzian function, the probe length increases quadratically with the focus distance.

The averaging of the wind speed over the probe length has the effect of a low pass filter on the wind speed power spectrum, which results in filtering out the turbulent structures with a length scale smaller than the probe length. Indeed, assuming the beam to be in the direction of the mean wind speed, the counterpart in the wave-number domain of the line-of-sight spatial averaging function (φ) is:

$$L_{Lorentzian} = e^{-2z_R|k_1|} \qquad (3)$$

where k_1 is the wave number along the mean wind direction. Sjöholm et al. [2] validated the effect of this filter by analysing spectra from wind speed measurements with a staring cw LiDAR and comparing them to sonic measurements.

If however the line-of-sight of the lidar does not coincidence with the mean wind direction, the ratio between the volume-averaged lidar-observed wind speed spectrum and the

---

1 The theoretical value is given by $z_R = \frac{\lambda\, r^2}{\pi a_0^2}$ where λ is the wavelength and a_0 is the radius in the telescope where the intensity has fallen to e^{-2} of its centre value. With a standard ZephIR's optical 3" lens design $a_0 \approx 24$ mm and hence theoretically should be $z_R \simeq 0.00085r^2$. However, our measurements have shown that the effective aperture is somewhat smaller, resulting in a best estimate formula: $z_R \simeq 0.0013r^2 = 0.0017z^2$

spectrum of the corresponding wind speed component measured by a sonic anemometer, will in general then be dependant on the entire three-dimensional structure of the turbulence [2].

In this paper we are considering the wind speed standard deviation (see section 3), therefore the integral of the spectrum and this does not depend on the shape of the spectra. We expect that the spectrum measured with a laser beam not aligned with the wind direction to have a different shape but the same integral.

2.2 Conically scanning lidar_ Spatial averaging over a circle

In order to get a 3 dimensional wind vector at a given height, radial wind speeds in different direction are required. To do so the QinetiQ ZephIR lidar conically scans the air, with a fixed angle Φ=30° from the vertical (see Figure 1), at the rate of one rotation per second. The cw lidar design allows a Doppler spectra acquisition frequency of 200000Hz. Each group of consecutive 4000 Doppler spectra are averaged (corresponding to an average over 20 milliseconds or a conical segment of 7.26m when focusing at 100m) in order to improve the signal to noise ratio, after which the Doppler peak should stand clearly above a flat shot-noise floor [7]. A radial wind speed is deduced from each averaged spectrum by finding the peak frequency (f_{peak}) using the centroid method. The radial speed (v_r) is directly proportional to the peak frequency according to the following relation:

$f_{peak} = \frac{2v_r}{\lambda}$ where λ is the laser wavelength. Nominally, 50 radial wind speeds are obtained for each rotation.

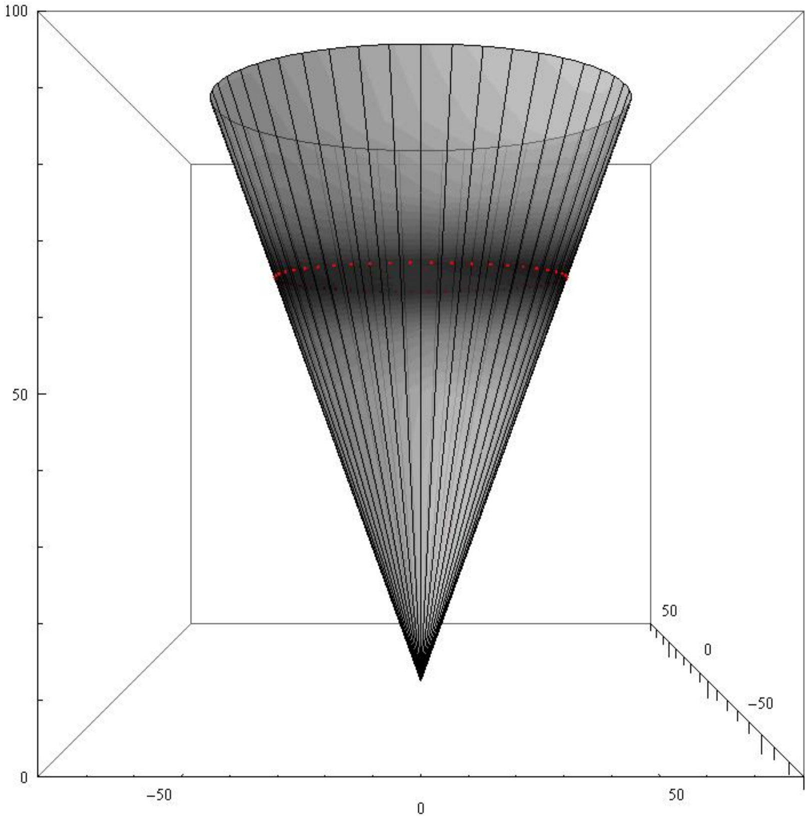

Figure 1 Sketch of the conical scanning pattern of the laser beam of a Zephir lidar. The gray scale represents the distribution (φ) of the power over the laser beam. Here the laser focuses at 70m agl. The red dots represent the middle of each 7.2° angle over which the Doppler spectra are averaged.

As the backscattered signal is mixed with the original signal, this lidar system cannot distinguish the sign of the Doppler shift (i.e. whether the wind comes towards the lidar or goes away). Therefore the 50 or 150 radial wind speeds as function of the azimuth angle follow a rectified cosine wave:

$$Vr(\theta) = \left| A \bullet Cos(\theta - \theta_{wind}) + B \right| \tag{4}$$

where θ is the beam azimuth angle (see Figure 2). We can then deduce the wind direction: θ_{wind}, the horizontal and vertical components of the wind speed: $U = A / Sin(\Phi)$, $w = B / Cos(\Phi)$ where Φ is the conical angle.

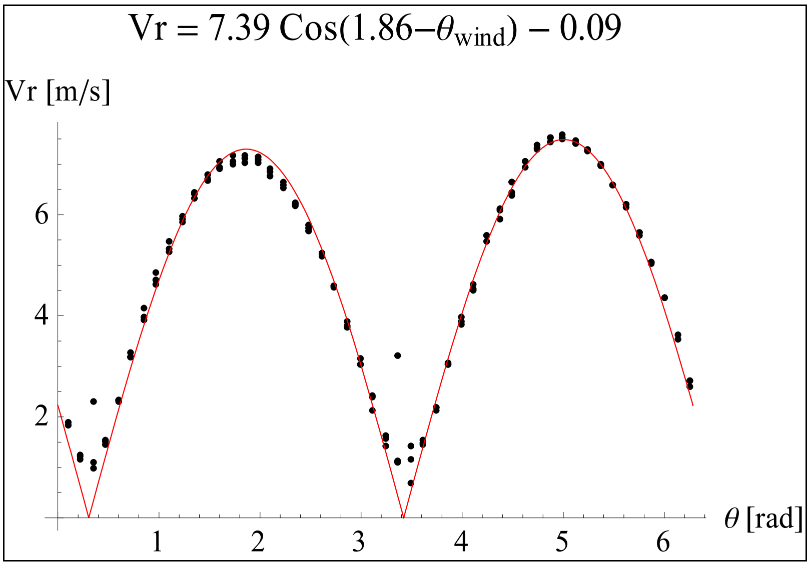

Figure 2 Example of result from a 3-second scan. The black points are the actual radial wind speed measurements. The red line is the fit function (given at the top of the figure).

Until spring 2008, the scanning time was fixed at 3 seconds (3 rotations for one wind speed vector). ZephiR lidars now offer the possibility to choose the scanning time of 1 second or 3 seconds (giving 50 or 150 radial wind speeds respectively). Both configurations are considered in this paper.

The first step of this investigation is to define a simple filter modeling the effect of the spatial averaging due to the scanning. An effective instantaneous horizontal averaging length scale can be estimated as the combined result of time lag and the circular coverage. For the ZephIR lidar, this means the distance covered by the circle of diameter *D* convected by the mean wind speed *U*. Figure 3 displays the path of the laser seen by the wind field, i.e. the path of the laser in the coordinate system attached to the wind field of mean direction the x-axis and mean speed *U*.

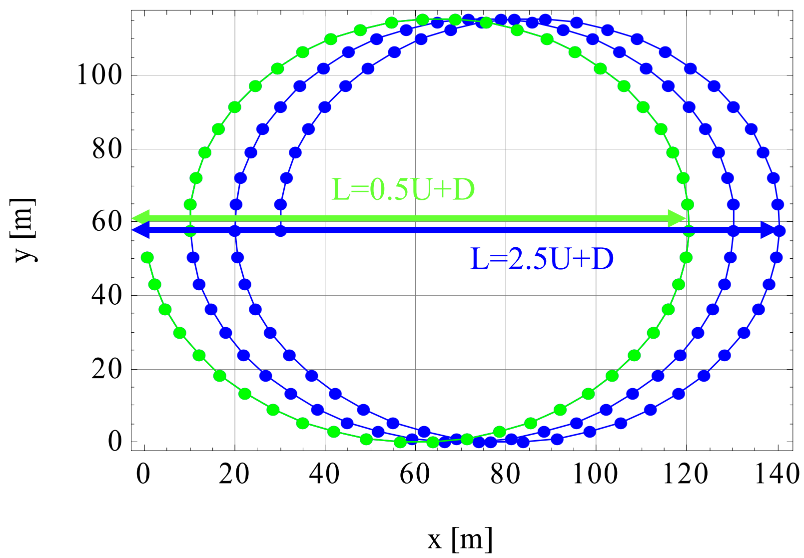

Figure 3 Path of the laser conically scanning at 50m high (then D=55m) and convected by the mean wind speed U=10m/s, for 1 second scanning in green and 3 seconds in blue. From that path we can estimate a representative length scale for the filter due to the combined effect of mean wind advection and conical scanning.

Therefore, a simple effective horizontal length scale, l_{az}, representing an effective filter-averaging length scale from conically scanning for n seconds ($n=1$ or 3), can to a first approximation be modelled by:

$$\begin{aligned} l_{az} &= D + U(n-0.5) \\ &= \frac{1}{Cos(30)} z + (n-0.5)U \qquad (5) \\ &= \frac{2}{\sqrt{3}} z + (n-0.5)U \end{aligned}$$

where z is the vertical distance to the ground where the measurement is taken (see Figure 3).

The conical scanning has also a low pass filter effect on the wind speed spectrum. Turbulence structures smaller than the length scale l_{az} cannot be effectively measured by the conically scanning lidar. As the magnitude of the radial wind speed varies with the azimuth angle following a rectified cosine function (see Figure 2), the horizontal components of turbulence are not filtered homogeneously over the length scale l_{az} , see Figure 4. In the stream wise direction, the weighting function can be modeled by a rectified sine function:

$$f_u(x) = \begin{cases} \left| \sin\left(\frac{\pi x}{l_{az}}\right) \right| & \text{if } |x| \le l_{az}/2 \\ 0 & \text{otherwise} \end{cases} \qquad (6);$$

The power spectral transfer function corresponding to (6) is obtained by normalising and squaring the Fourier transform in the wave-number domain:

$$L_{Scan}^{u} = \left[\frac{\pi}{4} \frac{1-\cos\left(\left(\frac{\pi}{l_{az}} - k\right)\frac{l_{az}}{2}\right)}{\left(\frac{\pi}{l_{az}} - k\right)\frac{l_{az}}{2}} + \frac{1-\cos\left(\left(\frac{\pi}{l_{az}} + k\right)\frac{l_{az}}{2}\right)}{\left(\frac{\pi}{l_{az}} + k\right)\frac{l_{az}}{2}} \right]^2 \qquad (7);$$

Similarly, in the cross stream direction, the weighting function can be defined by:

$$f_v(x) = \begin{cases} \cos\left(\dfrac{\pi x}{l_{az}}\right) & \text{if } |x| \le l_{az}/2 \\ 0 & \text{otherwise} \end{cases} \qquad (8);$$

giving in the following power spectral transfer function in the wave-number domain:

$$L_{Scan}^{v} = \left[\frac{\pi}{4} \frac{\sin\left(\left(\dfrac{\pi}{l_{az}} - k\right)\dfrac{l_{az}}{2}\right)}{\left(\dfrac{\pi}{l_{az}} - k\right)\dfrac{l_{az}}{2}} + \frac{\sin\left(\left(\dfrac{\pi}{l_{az}} + k\right)\dfrac{l_{az}}{2}\right)}{\left(\dfrac{\pi}{l_{az}} + k\right)\dfrac{l_{az}}{2}} \right]^2 \qquad (9)$$

In the vertical direction, all variation are taken into account homogeneously (Figure 5), therefore we can model this effect with a simple “Box car-like” filter function:

$$f_w(x) = \begin{cases} 1/l_{az} & \text{if } |x| \le l_{az}/2 \\ 0 & \text{otherwise} \end{cases} \qquad (10)$$

The corresponding power spectral transfer function in the wave-number domain is:

$$L_{Scan}^{w}(k_1) = \left[\frac{\sin\left(k_1 \dfrac{l_{az}}{2}\right)}{k_1 \dfrac{l_{az}}{2}} \right]^2 \qquad (11)$$

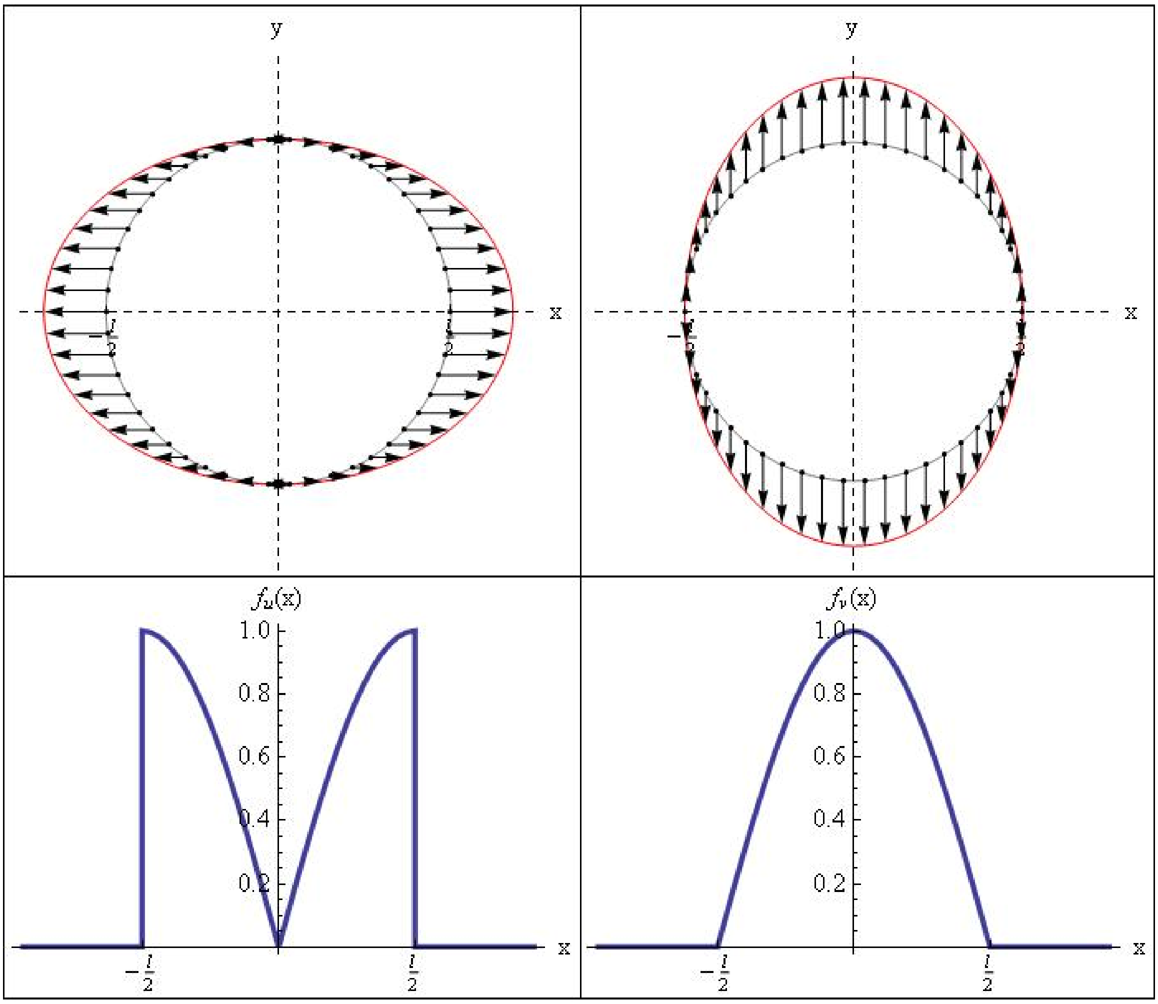

Figure 4 Top left and right: Distribution of the radial wind speed in the wind direction and the cross wind direction respectively (plan view from top of the cone); Bottom left and right: Weighting functions for the wind stream component and cross stream component of turbulence respectively.

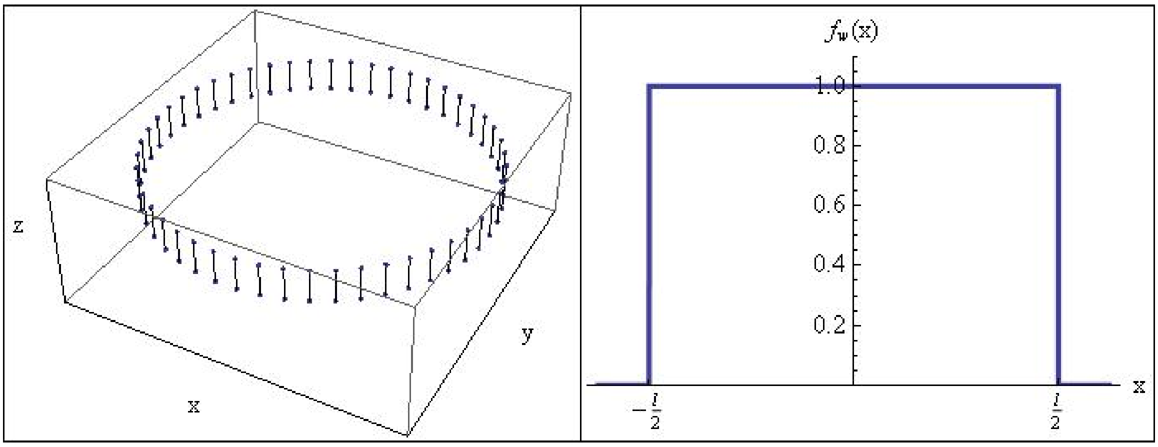

Figure 5 Left: Distribution of the radial wind speed in the vertical direction; Right: Weighting function for the vertical component of turbulence.

3 Lidar turbulence measurement

3.1 What can a cw conically scanning lidar tell us about turbulence?

A ZephIR can measure two different quantities that give information about turbulence. The first one is the wind speed standard deviation over 10 minutes, of the 1 or 3 seconds horizontal wind speeds (obtained from the fitting of the radial speeds). From measurements we know that the standard deviation measured by a LiDAR usually gives only about 80% of the standard deviation given by a cup anemometer, see Figure 6. This is mainly due to the volume averaging described in section 2. The standard deviation given by a conically scanning LiDAR does not effectively resolve the part of the turbulence having a scale smaller than the length l_{az} defined previously.

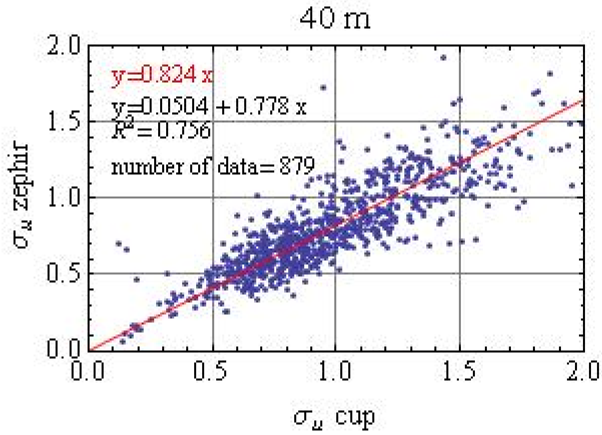

Figure 6 Example of regression plot of the 10 minutes wind speed standard deviation measured simultaneously by a Zephir LiDAR (3 second scan) and a cup anemometer at 40 m.

The second quantity provided by a ZephIR LiDAR is the so-called "turbulence parameter" (*TP*), defined as the turbulence intensity of the radial wind speed within the scanned circles:

$$TP_{1sec} = \frac{\sqrt{\left\langle v_r'^2 \right\rangle_{2\pi}}}{u_{1sec}} \qquad (12)$$

where $\left\langle v_r'^2 \right\rangle_{2\pi}$ is the variance of the radial wind speed over 1 rotation (2π) (see appendix) or for 1 second. We can define a similar quantity for a 3 second scan, then we average over 3 rotations: $\left\langle v_r'^2 \right\rangle_{6\pi}$. *TP* actually quantifies the goodness of fit of the rectified cosine function to the radial wind speeds measured over the circular path. This parameter contains information about the turbulence structures with a length scale smaller than l_{az} but larger than the laser beam radial probe length.

3.2 Illustration with the Kaimal spectrum

To evaluate the effect of spatial filtering we use the Kaimal spectrum transformed to wave number space. The transformation between the measured frequency and the wave number is based on the Taylor's frozen-wave hypothesis. So that

$$\sigma_i^2 = \int_0^{k_{1,\max}} F_i(k_1)dk_1 \qquad (13)$$

The stream-wise component of the Kaimal spectra in wave number space is then [4]:

$$F_u(k_1) = u_*^2 \frac{z}{2\pi} \frac{102}{(1 + 33\frac{k_1 z}{2\pi})^{5/3}} \left[\frac{m^3}{s^2}\right] \qquad (14)$$

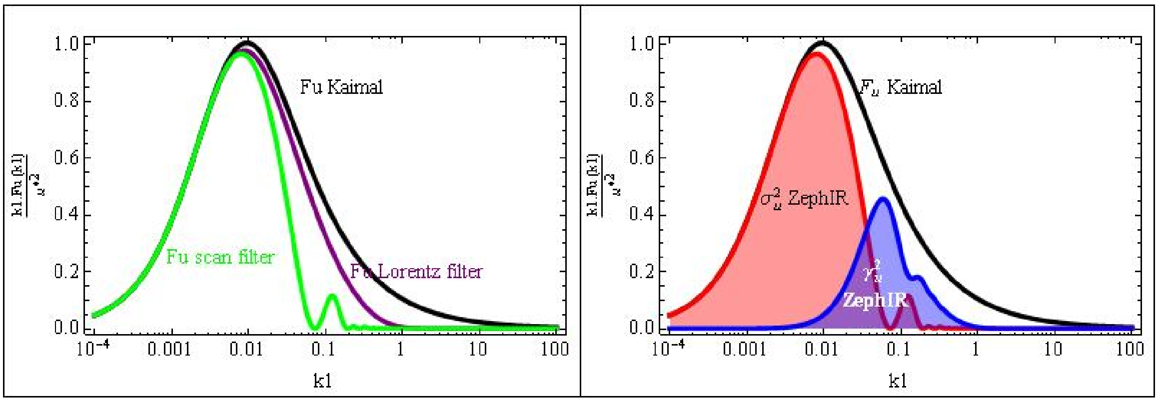

Figure 7 Stream wise component of the Kaimal spectrum (black) shown as a reference.

Left: spectrum filtered with the Lorentzian filter (purple), spectrum filtered with the scan filter (green). Right: spectrum of 10 minutes stream-wise turbulence obtained from a cw conically scanning LiDAR (red), turbulence spectrum obtained with the high-pass filter modeling the turbulence unresolved by the conical scanning effect (blue), see equation (17).The spectra shown correspond to a focus height z=30m *(then the circle diameter is 34.6m), for a 3-second scan and* $z_r=0.0017z^2$ *(*z=rCos(30°)*).*

Figure 7 (left) shows the effect of each spatial filter ($L_{Lorentzian}$ and L_{scan}) on the stream-wise component of the Kaimal spectrum. Both kinds of spatial averaging (over the probe volume and the scanning area) act as low pass filter removing all the small turbulence structures (high wave numbers). As the effect of the conical scanning is much stronger than the probe length averaging, the combination of both effects (red spectrum in right-handed plot in Figure 7) results in a low pass filter very close to the scanning filter (green spectrum in left-handed plot in Figure 7).

Firstly, Figure 7 (right) shows also the first component of the Kaimal spectrum as a reference. Its integral corresponds to the wind speed variance:

$$\sigma_u^2 = \int_0^{k_{1,\max}} F_u(k_1)dk_1 \qquad (15)$$

Secondly, Figure 7 (right) shows the spectrum obtained after applying the combination of both pass filters (red). For an ideal wind speed field experiencing only horizontal fluctuations and no vertical fluctuations, then the integration of this spectrum corresponds to the variance measured by a cw conically scanning LiDAR :

$$\left\langle u'^2 \right\rangle_{10min}^{ZephIR} = \sigma^2_{u_{ZephIR}} = \int_0^\infty F_u\left(k_1\right) L^u_{Scan}(k_1) \;\; L_{Lorentzian}(k_1)\; dk_1 \qquad (16)$$

Only large turbulence structures (with a length scale larger than l_{az}) are left. In reality, both the horizontal and the vertical speeds contribute to the radial wind speed and therefore, the wind speed standard deviation obtained with a LiDAR cannot be derived from the stream-wise turbulence spectrum only. However, our objective in this investigation is to make a simple model to comprehend basically the spatial averaging made by a cw conically scanning LiDAR. So we decided to make the rough assumption that the standard deviation from the LiDAR was coming only from the stream wise turbulence, i.e. we assume that equation (16) is also true for 3-D turbulence.

The third spectrum (blue) showed in Figure 7 was obtained by applying the low pass filter corresponding to the Lorentzian function and the high pass filter complementary to the scanning filter:

$$\left\langle \left\langle u'^2 \right\rangle_{2\pi}^{ZephIR} \right\rangle_{10min} = \int_0^\infty F_u\left(k_1\right) (1 - L^u_{Scan}(k_1)) \;\; L_{Lorentzian}(k_1)\; dk_1 \qquad (17)$$

This integral corresponds to the variance of the radial wind speed (as defined in the appendix) due to the stream-wise component of turbulence. As it is an average over 1 (or 3) second, it corresponds to the turbulence structures with a length scale smaller than l_{az} but larger than the length of the probe length ($2z_r$).
Since *TP* is a turbulence quantity characteristic of the small eddies, *TP* is a combination of the high wave number part of the spectra in all 3 dimensions:

$$\begin{aligned}\left\langle \left(TP_{1sec} \times u_{1sec}\right)^2 \right\rangle &= \left\langle \left\langle v_r'^2 \right\rangle_{2\pi}^{ZephIR} \right\rangle_{10min} \\ &= \frac{1}{2}\left(\frac{1}{2}\left\langle \left\langle u'^2 \right\rangle_{2\pi}^{ZephIR} \right\rangle_{10min} + \frac{1}{2}\left\langle \left\langle v'^2 \right\rangle_{2\pi}^{ZephIR} \right\rangle_{10min} + \frac{1}{4}\left\langle \left\langle w'^2 \right\rangle_{2\pi}^{ZephIR} \right\rangle_{10min} \right)\end{aligned} \qquad \mathbf{(18)}$$

$\left\langle \left\langle v'^2 \right\rangle_{2\pi}^{ZephIR} \right\rangle_{10min}$ and $\left\langle \left\langle w'^2 \right\rangle_{2\pi}^{ZephIR} \right\rangle_{10min}$ are obtained as $\left\langle \left\langle u'^2 \right\rangle_{2\pi}^{ZephIR} \right\rangle_{10min}$ in equation (17) but with the v and w component of the turbulence spectrum and with the corresponding filter definitions: L^v_{scan} and L^w_{scan}.

4 Experiment

4.1 Description of the experiment

Combined lidar and mast measurements were made at Risø DTU's Test Station for Large Wind Turbine in Høvsøre, western Denmark. Høvsøre is located in very flat terrain and is therefore an ideal site to make lidar measurement tests. The wind is mainly blowing from west (from the sea). The facility comprises an intensively instrumented meteorological mast. It has top mounted anemometer at 116.5m and measuring stations with both cup and 3-D ultrasonic anemometers at 100m, 80m, 60m and 40m. Due to its height, intensity of instrumentation and high data quality, this mast is well suited for testing lidar profilers. Cup anemometers are mounted on the south and sonic anemometers on the north of the mast. To compare lidar measurements to anemometer measurements we selected two wind sectors where the anemometers are not influenced by the mast: 240°-300° and 60°-120°.

The experimental data used for this investigation were obtained with three ZephiR lidars. The first one was equipped with the standard software using three seconds scanning to calculate the wind speed vector. The two others had an upgraded version of the software, giving the possibility of one second scanning. All lidar systems were positioned about 50m north of the met. mast (see sketch) in order to be far enough from the mast to avoid most of the mast wake effects, but also close enough to get sensible comparison with the mast instruments.

The data were selected for rain free periods. Moreover, a ceilometer placed nearby the mast enables us to measure the height of the cloud base. Because Zephir LiDAR measurements can be biased by low clouds, the periods with cloud base below 1600m were removed from the data sets used for the comparison.

The first data set (3 seconds scan) was measured in fall 2007. The LiDAR was measuring at 40m, 80m, 100m, 116.5m corresponding to mast measuring height. Therefore we could make comparison with the measurements from the sonic anemometers at 40m, 80m, and 100m. The second data set (1 second scan) was obtained in spring 2008. The lidar was measuring at 60m, 80m, 100m and 116.5m. However a faulty sonic anemometer at 80m prevented us from making comparisons at that height. Therefore we have results at only 2 heights: 60m and 100m. The third dataset (1 second scan) was obtained in February 2009. This lidar was set to measure at only one height: 80m.

As turbulence measurements can be easily corrupted by noise, we selected the 10 minutes period when the average number of points used to calculate the fit function was higher than 65% of the maximum possible number of points.

4.2 Results

In order to observe the effect of the spatial averaging, we look at the ratio between measurements from the two instruments: lidar and anemometer. For both the wind speed standard deviation and the radial speed variance, the model is compared to experimental data at different heights, thus we can observe the variation of the effect of spatial averaging with height. Moreover, as the length scale l_{az} depends on the number of rotations (n) realized during the scanning, we show the results for 1 and 3 revolutions in different plots.

1) Standard deviation:

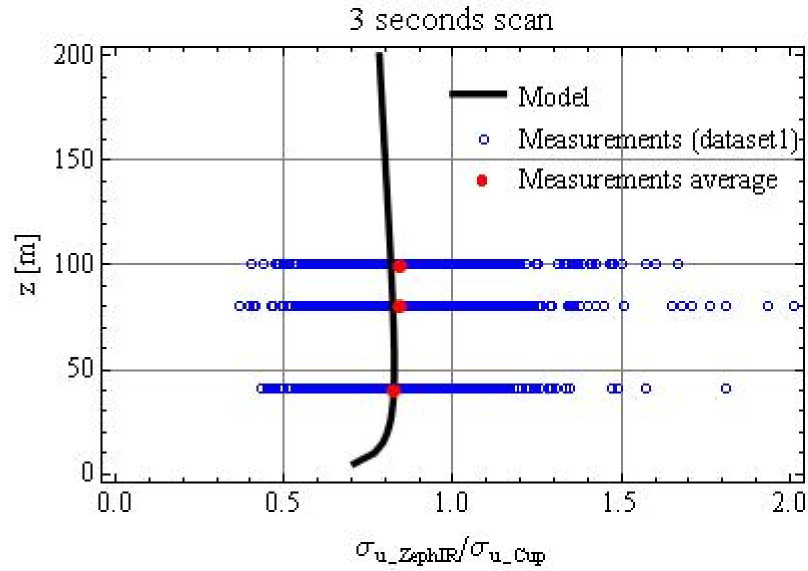

Figure 8 Variation with height of the ratio between wind speed standard deviation measured by the Zephir lidar scanning for 3 seconds (3 rotations for each wind speeds vector) and the wind speed standard deviation measured by the cup anemometer. Comparison of the model presented in part 3 and experimental data.

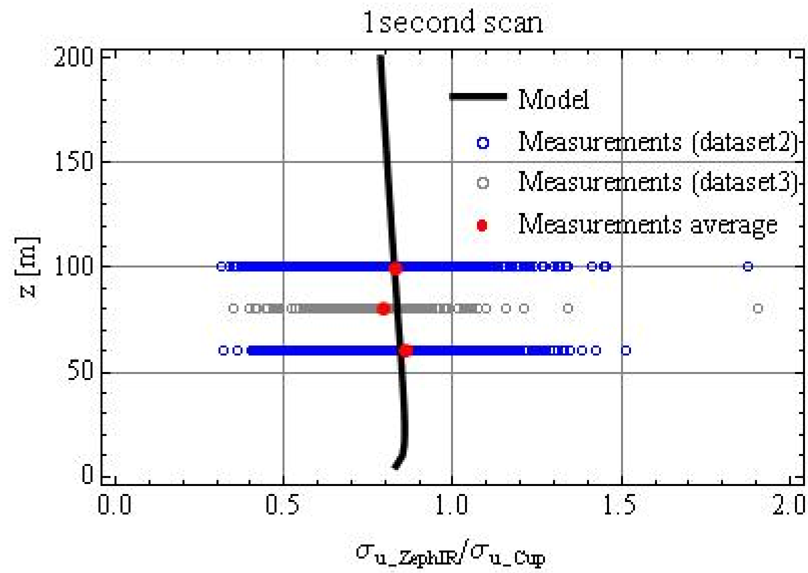

Figure 9 Variation with height of the ratio between wind speed standard deviation measured by the Zephir lidar scanning for 1 second (1 rotation for each wind speeds vector) and the wind speed standard deviation measured by the cup anemometer. Comparison of the model presented in part 3 and experimental data.

Comparison of the wind speed standard deviation measured by a ZephIR LiDAR to cup anemometers measurements were performed in several places, with different ZephIR LiDAR units in various types of terrain. Figure 10 sum up the results for 8 tests in addition to the measurements we made in Høvsøre.

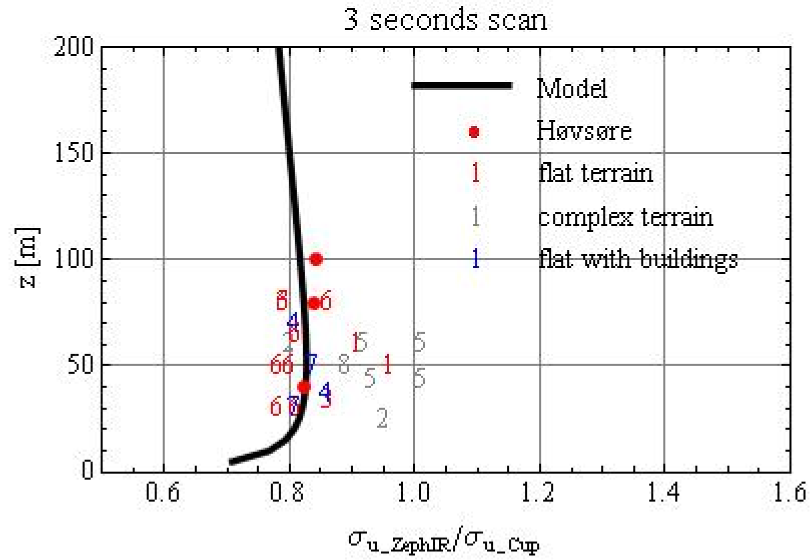

Figure 10 Comparison of the ratios between wind speed standard deviation measured by a ZephIR LiDAR and a cup anemometer for 8 experiments made in various types of terrain (flat, flat with building, complex). The numbers displayed in the plot corresponds to the experiment number and the color to the type of terrain.

2) Radial speed variance

In the follwing plots, we are comparing the quantities:

$$\left\langle v_r^{'2} \right\rangle_{ZephIR} = \left\langle \left(TP_{1sec} \times u_{1sec} \right)^2 \right\rangle = \left\langle \left\langle v_r^{'2} \right\rangle_{2\pi}^{ZephIR} \right\rangle_{10min}$$

and

$$\left\langle v_r^{'2} \right\rangle_{sonic} = \left\langle v_r^{'2} \right\rangle_{10min}^{sonic} = \tfrac{1}{2}\left(\tfrac{1}{4}\left\langle u^{'2} \right\rangle_{10min}^{sonic} + \tfrac{1}{4}\left\langle v^{'2} \right\rangle_{10min}^{sonic} + \tfrac{3}{2}\left\langle w^{'2} \right\rangle_{10min}^{sonic} \right)$$

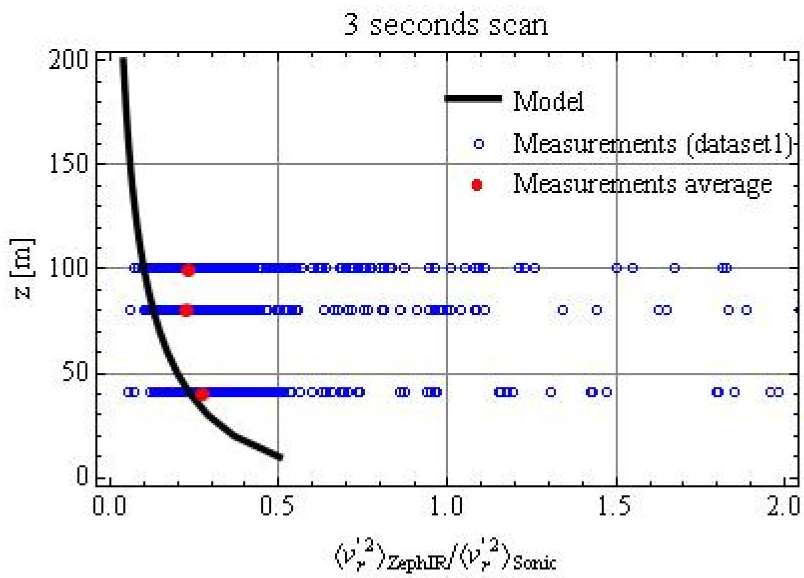

Figure 11 Variation with height of the ratio between radial speed variance measured by the Zephir lidar scanning for 3 seconds (3 rotations for each wind speeds vector) and the radial speed variance measured by the 3-D sonic anemometer. Comparison of the model presented in part 3 and experimental data.

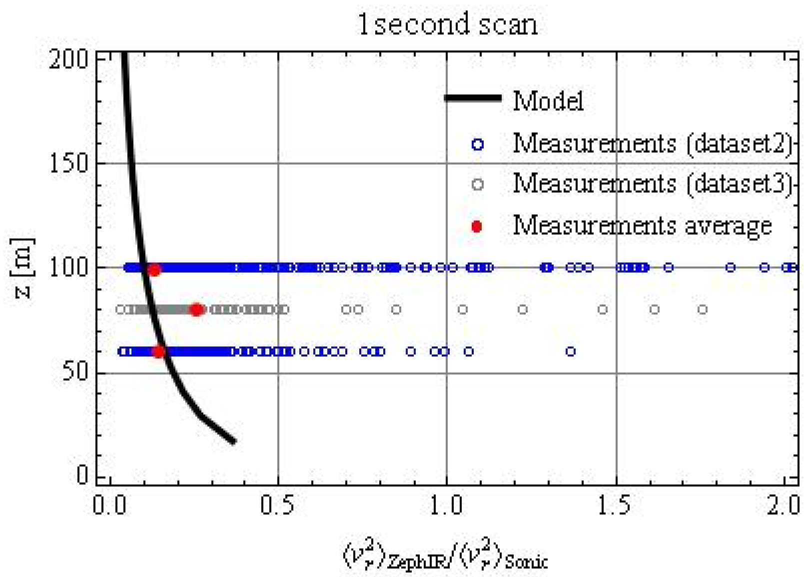

Figure 12 Variation with height of the ratio between radial speed variance measured by the ZephIR lidar scanning for 1 second (1 rotation for each wind speeds vector) and the radial speed variance measured by the 3-D sonic anemometer. Comparison of the model presented in part 3 and experimental data.

5 Discussion

5.1 Standard deviation

The first observation we can make from Figures 8, 9 and 10 is the wind speed standard deviation measured by a LiDAR gives on average about 80% of the standard deviation measured by a cup anemometer in flat terrain. The experimental data coincide with the prediction of the model, which favours the proposed model of turbulence filtering due to the spatial averaging. Moreover it confirms the fact that the volume averaging is the main reason of the difference of the wind speed standard deviation between a cw conically scanning LiDAR and an anemometer.

We can observe that the ratio between the standard deviation measured by the two instruments decreases with height. Indeed the averaging volume increases with height. Finally, the ratio is generally higher for the LiDAR scanning for 1second than for the LiDAR scanning for 3 seconds. Again, this is a direct consequence of the size of the averaging volume which is smaller for a one second scan than for a three seconds scan.

5.2 the radial speed variance

The radial speed variance is generally decreasing with height, see Figures 11 and 12. This is due to a combination of different factors including the increasing spatial volume of averaging and the decreasing amount of small turbulence structures with height.

For both cases (1 sec and 3 sec scanning) we get good agreements between the model and the measurements at the lowest heights (40m and 60m) but not that good at higher heights. A possible explanation is that TP depends on the goodness of fit of the rectified cosine function to the radial speed points which implies that TP also accounts for measurement errors including noise, and not only for turbulence. As the amount of small turbulence structure decreases with height, $\left\langle v_r^2 \right\rangle$ might account for errors and noise more than turbulence at high heights (error that are not taken into account in the model).

6 Conclusions

We defined a volumetric filtering of the turbulence to represent the effect of the spatial averaging operated by a lidar when measuring the wind speed. As the objective was to understand the principle of the spatial averaging, we decided to keep the model simple, which implied that we have made some rough approximations. Nevertheless, the averaged experimental data coincide relatively well with the model. This confirms that the difference in wind speed standard deviation measured by a lidar and a cup anemometer is mainly due to the spatial averaging.

In average, a cw conically scanning lidar measures 80% of the standard deviation measured by an anemometer, because such a lidar cannot resolve the turbulence structures with a length scale smaller than the circle (described by the scanning lidar) diameter approximately. Meanwhile, the turbulence parameter given by a ZephIR lidar can give some information about the smaller eddies. However, as it also quantifies the goodness of fit of the radial wind speed when calculating the wind speed vector, the turbulence parameter is easily corrupted by noise and it should be considered only as an average indicator of the high frequency turbulence.

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Appendix: definition of the variance of the radial wind speed

Referred to a spherical coordinate system the radial wind speed can be expressed as

$$V_r(\theta) = \left| u \sin\varphi \cos\theta + v \sin\varphi \sin\theta + w \cos\varphi \right| \quad (19)$$

where u, v and w are the wind components of the instantaneous wind vector, θ is the beams azimuth angle and φ the conical angle. For the configuration of the QinetiQ ZephIR wind lidar, θ is scanning over the interval [0; 2π] and $\varphi = 30^\circ$. From this, (10) becomes

$$V_r(\theta) = \left| \tfrac{1}{2} u \cos\theta + \tfrac{1}{2} v \sin\theta + \tfrac{\sqrt{3}}{2} w \right| \quad (20)$$

We next denote the fit function to the radial speeds as function of the azimuth angle (θ), assuming that the vertical mean wind speed over a revolution is null, as

$$v_{r\_fit}(\theta) = \tfrac{1}{2}\langle u \rangle_{2\pi} \cos\theta + \tfrac{1}{2}\langle v \rangle_{2\pi} \sin\theta \quad \textit{(21)}$$

where $\langle\ \rangle_{2\pi}$ denotes an average over 1 full revolution ($\langle x \rangle_{2\pi} = \frac{1}{2\pi}\int_0^{2\pi} x \, d\theta$).

By the usual Reynolds decomposition (u*(t)*=<*u*>+u') we can now, by subtracting (13) from (12), obtain the following expression for the fluctuations in the radial measured wind speed component

$$\begin{aligned} v_r^{'}(\theta) &= v_r(\theta) - v_{r\_fit}(\theta) \\ &= \tfrac{1}{2} u^{'} \cos\theta + \tfrac{1}{2} v^{'} \sin\theta + \tfrac{\sqrt{3}}{2} w^{'} \end{aligned} \quad \textit{(22)}$$

where $v_r^{'}$ denotes the instantaneous radial velocity component, as function of azimuth θ.

By squaring, and subsequently taking the average over one full revolution (index 2π), we finally obtain

$$\begin{aligned} \langle v_r^{'2} \rangle_{2\pi} &= \left\langle \left(\tfrac{1}{2} u^{'} \cos(\theta) + \tfrac{1}{2} v^{'} \sin(\theta) + \tfrac{\sqrt{3}}{2} w^{'} \right)^2 \right\rangle_{2\pi} \\ &= \tfrac{1}{4}\langle u^{'2} \rangle_{2\pi} \langle \cos^2(\theta) \rangle_{2\pi} + \tfrac{1}{4}\langle v^{'2} \rangle_{2\pi} \langle \sin^2(\theta) \rangle_{2\pi} + \tfrac{3}{4}\langle w^{'2} \rangle_{2\pi} \\ &= \tfrac{1}{4}\langle u^{'2} \rangle_{2\pi} \tfrac{1}{2} + \tfrac{1}{4}\langle v^{'2} \rangle_{2\pi} \tfrac{1}{2} + \tfrac{3}{4}\langle w^{'2} \rangle_{2\pi} \\ &= \tfrac{1}{2}\left(\tfrac{1}{4}\langle u^{'2} \rangle_{2\pi} + \tfrac{1}{4}\langle v^{'2} \rangle_{2\pi} + \tfrac{3}{2}\langle w^{'2} \rangle_{2\pi} \right) \end{aligned} \quad \textit{(23)}$$

Because of the azimuth averaging, all of the cross covariance terms $\langle u'w' \rangle_{2\pi}, \langle v'w' \rangle_{2\pi}, \langle u'v' \rangle_{2\pi}$ will drop out. Therefore $\langle v_r^{'2} \rangle$ for a LiDAR corresponds to the average over 10 minutes of the quantity $\langle v_r^{'2} \rangle_{2\pi}$; for a sonic anemometer it is calculated directly from the 10 minutes variance of *u*, *v* and *w*.