Structural optimisation of a radial-flux permanent magnet generator (for a direct-drive wind turbine) using a Genetic Algorithm

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1. Introduction

This report follows on from "Development of analytical tools for estimating inactive mass" [1] which produced analytical tools for estimating the structural mass required in large, low-speed electrical generators for large offshore direct-drive wind turbines. This work (also detailed in [2]) linked the mechanical design to the electromagnetic design, so that these generators could be designed and optimised. Some examples were given showing how the optimising the electromagnetically active material in isolation can lead to non-optimal (in terms of weight) solutions as compared to this integrated approach. In this report a more formal optimisation process will be carried out on a generator rotor structure using a Genetic Algorithm.

2. Genetic Algorithm

The MATLAB Genetic Algorithm Toolbox is a freely downloadable toolbox developed by the Evolutionary Computation Research Team at the University of Sheffield [3], [4]. A Genetic Algorithm is an optimisation process inspired by biological evolution. This "off-the-shelf" code was chosen because of its ease of use and good documentation – the novelty is in its application described herein. The toolbox can be tailored to each problem by writing an objective function which evaluates how good a design is and returns a rating – the toolbox essentially does all the rest of the work.

3. Rotor structure with arms

This report examines a rotor structure with arms. In the first part the aim was to try and find the minimum mass for a feasible structure. The main criterion for the structure to be successful is that the radial deflection is not too large; later axial and circumferential deflection constraints were also introduced.

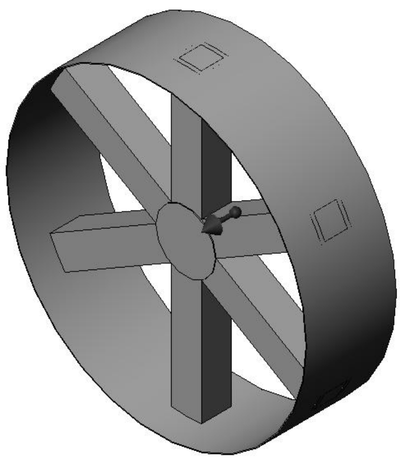

Fig. 1. Rotor structure with six arms.

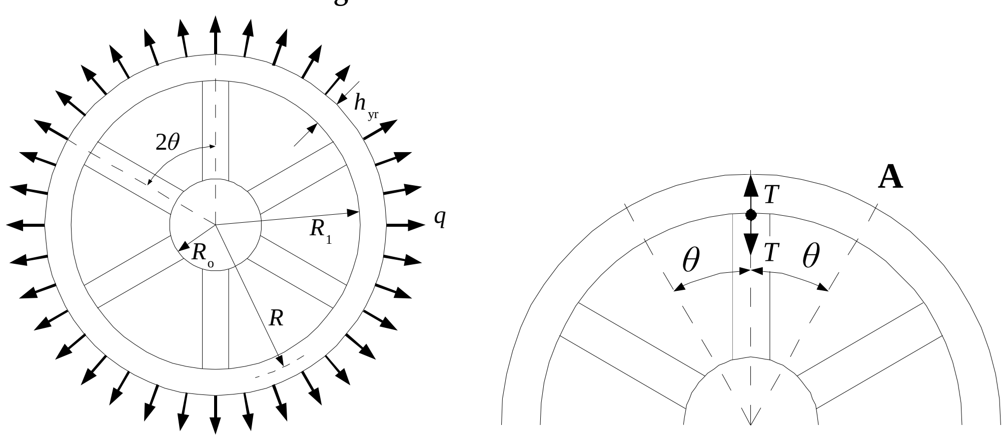

Fig. 2. Rotor structure with arms: dimensions.

3.1 Variables

There are a number of structural variable which describe the rotor. The independent variables are listed first and then the dependent variables. These are shown in Fig. 1 and 2.

3.1.1 Independent variables

n number of arms
t thickness of rotor cylinder back, m
b arm dimension, circumferential, m
d arm dimension, axial, m
t_w wall thickness of arm, m. (This can be seen in Fig. 3).

The following two variables may be set as constants for simplification; later a penalty is used in the objective function to ensure that the machine is of a suitable size.

R rotor cylinder radius, m
l axial length of rotor cylinder, m

3.1.2 Dependent variables

θ half angle between spokes, radians

$$\theta = \frac{\pi}{n} \qquad (1)$$

I second moment of area of rotor cylinder, m^4

$$I = \frac{lt^3}{12} \qquad (2)$$

A cross sectional area of rotor cylinder, m^2

$$A = lt \qquad (3)$$

a cross sectional area of rotor arms, m^2

$$a = \left[bd - (b - 2t_w)(d - 2t_w)\right] \qquad (4)$$

R_1 inner radius of rotor cylinder, m

$$R_1 = R - \tfrac{1}{2}t \qquad (5)$$

k radius of gyration, m

$$k = \sqrt{\frac{I}{A}} \qquad (6)$$

m

$$m = \left(\frac{k}{R}\right)^2 \qquad (7)$$

M mass of structure, kg

$$M = 2\pi tlR\rho + n(R_1 - R_0)a\rho \qquad (8)$$

3.1.3 Analytical models

The rotor structure is assessed for fitness in three different ways: radial deflection (into the airgap due to the normal component of Maxwell stress), axial deflection (due to the weight of generator material when the machine's axis is vertically orientated – i.e. when it is being transported, assembled or lifted) and circumferential deflection (twist of the outside of the rotor relative to the rotor shaft due to the torque/shear stress).

u_{an} radial deflection found analytically, m. Equation (9) is taken from [2]:

$$u_{\text{an}} = \frac{qR^2}{Et}\left(1+\frac{R^3\left[\frac{(\sin\theta-\theta\cos\theta)}{4\sin^2\theta}-\frac{1}{2\sin\theta}+\frac{1}{2\theta}\right]}{I\left[\left(\frac{\theta}{\sin^2\theta}+\frac{1}{\tan\theta}\right)\left(\frac{R}{4A}+\frac{R^3}{4I}\right)-\frac{R^3}{2I\theta}\left(\frac{1}{m+1}\right)+\frac{R_1-R_o}{a}\right]}\right) \quad (9)$$

u_{allow} allowable radial deflection, m

$$u_{\text{allow}} = \frac{a_c}{20} = \frac{1}{20}\frac{(2R)}{1000} \quad (10)$$

The radial deflection is restricted to 5% of the airgap length (which is assumed to be 1% of the airgap diameter [5]).

y_{an} axial deflection, m.

$$y_{\text{an}} = y_{\text{a,i}} + y_{\text{a,ii}} \quad (11)$$

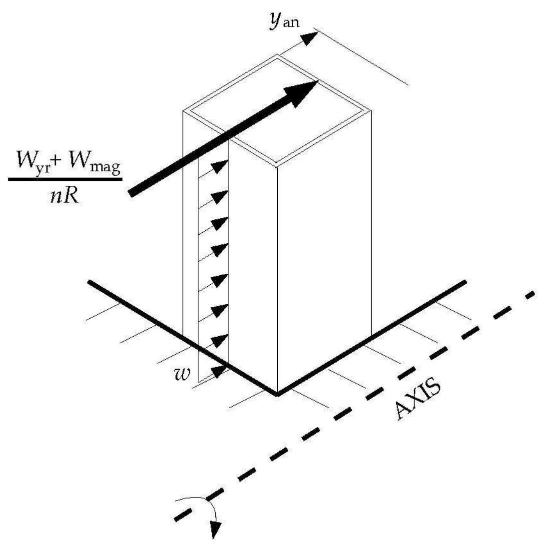

Fig. 3. Deflection due to axial weight.

$y_{\text{a,i}}$ axial deflection due to part of rotor cylinder weight, m

$$y_{\text{a,i}} = \frac{Wl_{\text{i}}^3}{12EI_{\text{arm,axi}}} \quad (12)$$

$y_{\text{a,ii}}$ axial deflection due to rotor arm weight, m

$$y_{\text{a,ii}} = \frac{wl_{\text{ii}}^4}{24EI_{\text{arm,axi}}} \quad (13)$$

y_{allow} allowable axial deflection, m. The allowable deflection is assumed to be 1% of the axial length:

$$y_{\text{allow}} = \frac{l}{100} \quad (14)$$

l_{i} length of (rotor arm) beam at which rotor cylinder acts, m

$$l_{\text{i}} = R \quad (15)$$

l_{ii} length of (rotor arm) beam on which self weight acts, m

$$l_{ii} = R_1 \qquad (16)$$

$I_{arm,axi}$ second moment of area of rotor arm (for weight), m^4

$$I_{arm,axi} = \frac{bd^3 - (b - 2t_w)(d - 2t_w)^3}{12} \qquad (17)$$

W weight of n^{th} of rotor cylinder, N. The tilt angle φ is 5-10^o for normal orientation, but is set to 90^o for this study to represent assembly, transportation and lifting operations.

$$W = (2\pi R / n)\rho g \sin(\varphi)lt \qquad (18)$$

w self weight uniformly distributed load of rotor arm, Nm

$$w = \rho g \sin(\varphi)a \qquad (19)$$

$I_{arm,tot}$ Second moment of area of rotor arm w.r.t. torsion, m^4

$$I_{arm,tor} = \frac{db^3}{12} - \frac{(d - 2t_w)(b - 2t_w)^3}{12} \qquad (20)$$

z_A Circumferential deflection, m. This is shown in Fig. 4.

$$z_{an} = \frac{(2\pi Rl/n)\sigma l_i^3}{3EI_{arm,tor}} \qquad (21)$$

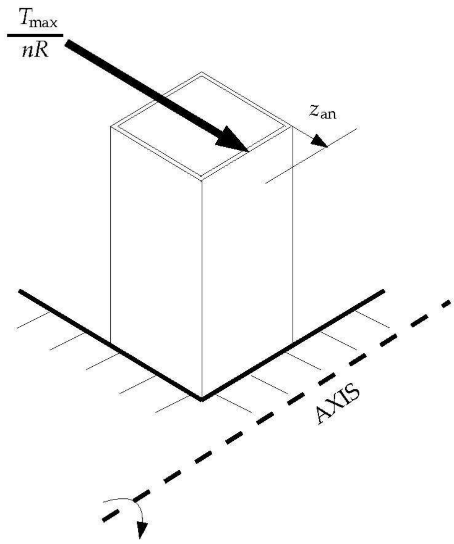

Fig. 4. Deflection due to shear force/torque.

z_{allow} allowable torsional deflection, m. It is assumed that the relative twist is 0.5^o. This is probably a very relaxed limit but further study of the structural dynamics and vibrations should lead to a stricter criterion which represents how stiff the structure should be in this direction so that it is not easily excited.

$$z_{allow} = \frac{0.5}{360} 2\pi R \qquad (22)$$

3.1.4 Constants

E Young's Modulus, 200 GPa

ρ density, 7850 kg/m^3

R_o radius of rotor shaft, 0.5m

q normal stress, 280 kPa

g acceleration due to gravity, 9.81m/s^2

3.1.5 Common penalties/constraints

The objective function is set up to penalise designs if:

(i) If $u_{an} > u_{allow}$. Interpreted as: *the radial deflection into the airgap is too large.*

(ii) If $y > y_{allow}$. Interpreted as: *the axial deflection is too large.*

(iii) If $z > z_{allow}$. Interpreted as: *the torsional deflection is too large.*

(iv) If $b > \frac{2\pi R_o}{n}$. Interpreted as: *the dimension* b *is greater than* nth *of the rotor shaft circumference*

(v) If $R^2 l < \frac{T}{2\pi\sigma}$. Interpreted as: *the dimensions* R *and* l *are too small in order to provide the necessary torque* T.

4. Optimisations for specific Aspect Ratio

The main of these optimisations is to find lightweight structures. In this case the aspect ratio, radius and axial lengths are chosen for a 5MW rotor [2]. The optimisation is carried out for each value of aspect ratio, K_{rad} (Table 1). The value returned by the rating function is of the form:

```
val =
((2*pi.*t.*l.*R*rho)+(N.*(R_1-R_o).*a*rho))      MASS
+((sign(u_A-u_all)+1).*(u_A-u_all).^3.*5e17)     RAD DEF
+(((sign(y_A-y_all)+1).*(y_A-y_all).^3.*5e17))   AXI DEF
+(((sign(z_A-z_all)+1).*(z_A-z_all).^3.*5e17))   CIR DEF
+(((sign(b-b_all)+1).*(b-b_all).^3.*5e13));      ARM WIDTH
```

So that structures which have deflection larger than allowed are penalised. There is also a constraint so that the width of the arms is restricted to the physical dimension at the rotor shaft radius.

The optimisation was carried out several times for each value of K_{rad} (the GA cannot guarantee that the best solution is found; just that generally the final solution is better than the starting values). The best solutions are shown in Table 1.

K_{rad}	**0.022**	**0.061**	**0.12**	**0.283**	**0.955**
R (m)	7.0	5.0	4.0	3.0	2.0
l (m)	0.31	0.61	0.96	1.7	3.82
Best Score:	36654	28218	25356	22130	19673
n	5.04	5.06	5.05	5.02	7.1
t (m)	0.181	0.129	0.103	0.0763	0.0473
b (m)	0.942	0.942	0.94	0.939	0.672
d (m)	0.962	0.769	0.578	0.649	0.768
t_w (m)	0.0192	0.0161	0.0159	0.0108	0.00974
Mass (kg)	**3.669×10^4**	**2.828×10^4**	**2.544×10^4**	**2.213×10^4**	**1.968×10^4**
u_A (m)	6.996×10^{-4}	4.987×10^{-4}	3.983×10^{-4}	3.000×10^{-4}	1.997×10^{-4}
y_A (m)	2.849×10^{-4}	1.157×10^{-4}	8.324×10^{-5}	2.235×10^{-5}	3.542×10^{-6}
z_A (m)	0.0061	0.0044	0.0035	0.0026	0.0017

Table 1. Best optimal results for a 5MW generator rotor structure with arms for a range of K_{rad}

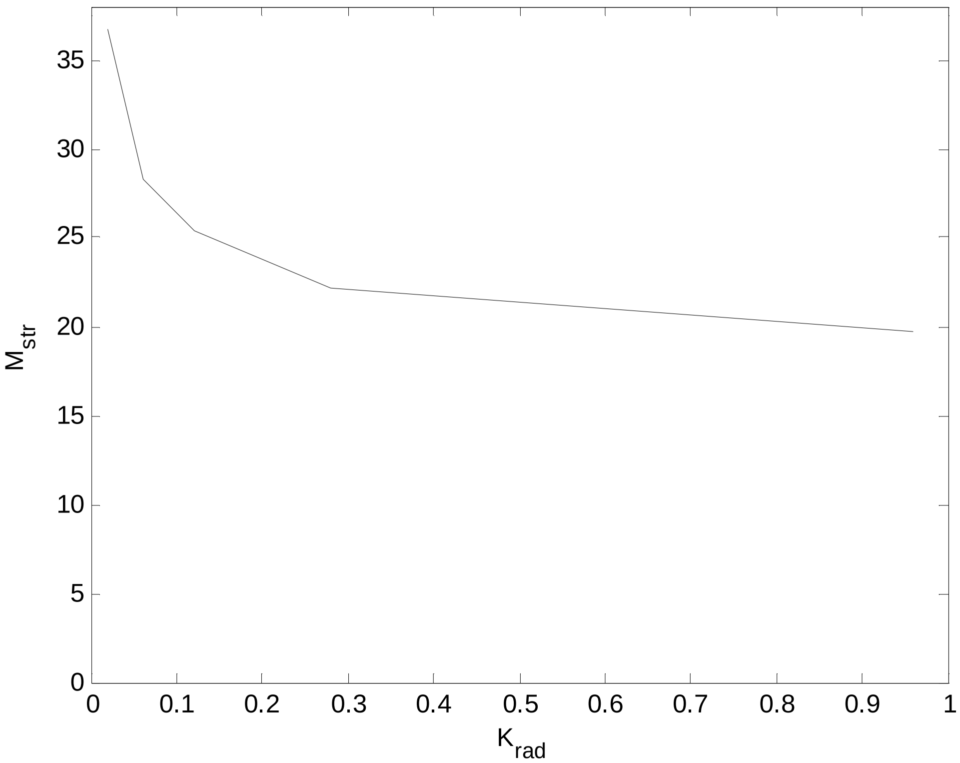

Fig. 5. Structural mass results (in 10^3 kg) for a 5MW generator rotor structure with arms for a range of K_{rad}

5. Optimisation with variable Aspect Ratio

In this section the aspect ratio is allowed to vary and the structure is optimised again for minimum mass. The value returned by the rating function is of the form:

```
val =
((2*pi.*t.*l.*R*rho)+(N.*(R_1-R_o).*a*rho))     MASS
+((sign(u_A-u_all)+1).*(u_A-u_all).^3.*5e17)    RAD DEF
+(((sign(y_A-y_all)+1).*(y_A-y_all).^3.*5e17))  AXI DEF
+(((sign(z_A-z_all)+1).*(z_A-z_all).^3.*5e17))  TOR DEF
+(((sign(b-b_all)+1).*(b-b_all).^3.*5e13))      ARM WIDTH
+((sign((T/2/pi/sigma)-(R.^2.*l))+1).*((T/2/pi/sigma)-
(R.^2.*l)).^3*5e10);                            TORQUE
```

The main difference between this formulation and the previous one is that there is a minimum torque constraint. This is achieved by assuming a shear stress, σ and ensuring that the inequality $R^2 l \geq \frac{T}{2\pi\sigma}$ is met.

Ranges for variables
R 1-9 m
l 0.2-5 m
n: 5-15
t 0.001-0.2 m
b 0.1-1.5 m
d 0.1-1.5 m
t_w 0.001-0.099 m

The optimal value returned (after many repeat runs) was a structural mass of 20398 kg.
R=2.14 m
l=3.3 m
n=5
t=0.0541 m
b=0.838 m
d=0.647 m
t_w=0.00991 m

Giving deflections:
$u_{an} = 2.14\times10^{-4}$ m ($u_{all} = 2.14\times10^{-4}$ m)
$y_{an} = 6.53\times10^{-6}$ m ($y_{all} = 0.0330$ m)
$z_{an} = 0.0019$ m ($z_{all} = 0.0019$ m)

Note that this leads to a larger mass than was found using the fixed aspect ratios. This is due to the dimensions in Table 1 not quite satisfying the inequality $R^2 l \geq \frac{T}{2\pi\sigma}$ (i.e. the result here is more reasonable). The optimum aspect ratio was found to be $K_{rad} = 0.77$.

6. Introducing electromagnetic material into the optimisation process

It is apparent when looking at actual directly-driven generators for large wind turbines that much smaller aspect ratios are usual. The optimisations so far have looked at the structural material in isolation. If the electromagnetically active material is considered then larger radius machines are more favourable (as this material – which is more expensive – can be reduced).

As a starting point the magnetic material for any given generator is modelled. The shear stress in the previous section is assumed to require an airgap flux density of B_g = 0.85 T.

$$B_g = B_r \frac{h_m/\mu_r}{h_m/\mu_r + c} \frac{4}{\pi} \sin\left(\frac{b_m}{\tau_p}\frac{\pi}{2}\right) \qquad (23)$$

Assuming also that $h_m/\mu_r \to h_m$, $\frac{b_m}{\tau_p}=0.8$, $c=\frac{R}{500}$, and $K=\frac{4}{\pi}\sin\left(\frac{b_m}{\tau_p}\frac{\pi}{2}\right)$ leads to

$$h_m = \frac{c}{B_r K} \frac{1}{\left(\left(\frac{1}{B_g}\right)-\left(\frac{1}{B_r K}\right)\right)}. \qquad (24)$$

Equation (24) allows the mass of magnetic material to be estimated for any radius (and hence airgap length c).

For example for a rotor radius R=1.92m, c=0.0038m. If the magnets have a remanent flux density of B_r=1.2T then to produce the airgap flux density B_g=0.85T a magnet height of $\boldsymbol{h_m}$**=0.0054 m** is required.

The mass of the magnets is included in the total mass:

$$mass_{PM} = 2\pi Rl\left(\frac{b_m}{\tau_p}\right)h_m\rho_{PM} \qquad (25)$$

Note too that W is altered thus:

$$W = g\sin(\varphi)\left[\left((2\pi R/n)\rho lt\right)+(mass_{PM}/n)\right], \qquad (26)$$

so that the axial deflection modelling also takes this into account.

```
val =
mass_PM+((2*pi.*t.*l.*R*rho)+(N.*(R_1-R_o).*a*rho)
+((sign(u_A-u_all)+1).*(u_A-u_all).^3.*5e22)
+(((sign(y_A-y_all)+1).*(y_A-y_all).^3.*5e17))
+(((sign(z_A-z_all)+1).*(z_A-z_all).^3.*5e17))
+(((sign(b-b_all)+1).*(b-b_all).^3.*5e13))
+((sign((T/2/pi/sigma)-(R.^2.*l))+1).*((T/2/pi/sigma)-
(R.^2.*l)).^3*5e10);
```

After several optimisation runs the minimal total mass was found to be **22294 kg**, which the following parameters:

R=2.06m
l=3.55m
n=6
t=0.0519m

b=0.771m
d=0.37m
t_w=0.0144m

The increased aspect ratio is at first surprising, K_{rad}=0.86 (one would think that the additional requirement to reduce magnet mass would tend to reduce the aspect ratio). The main reason is the dependence of the airgap length on the airgap radius (e.g. eqn. 10). As the aspect ratio increases, the radius and airgap length both increase. This has the effect of requiring that the magnet height increase in order to produce the same airgap flux density (which may outweigh the reduction in magnet volume from being at a larger radius, i.e. eqn (25)).

One can also include the steel in the rotor back in this calculation. The structural steel and the steel required for the magnetic circuit are assumed to be separate here (this may be the case for fractional-slot machines where the rotor steel should be laminated to reduce losses; for integral-slot machines the rotor back iron can be solid steel and t and h_{ry} are the same dimension). The mass of the steel is

$$mass_{\text{steel,ry}} = 2\pi R l h_{\text{ry}} \rho_{\text{steel}} \,. \qquad (27)$$

The height of the steel yoke, h_{ry} is given by:

$$h_{\text{ry}} = \tfrac{1}{2} \frac{B_{\text{r}}}{B_{\text{st}}} b_{\text{m}} \frac{h_{\text{m}}/\mu_{\text{r}}}{h_{\text{m}}/\mu_{\text{r}} + c} \,. \qquad (28)$$

Assuming that τ_p=0.1m and b_m=0.08m for all machines this mass can be included in the optimisation function and the axial weight calculation:

$$W = g\sin(\varphi)[((2\pi R/n)\rho l t) + (mass_{\text{PM}}/n) + (mass_{\text{steel,ry}}/n)] \qquad (26b)$$

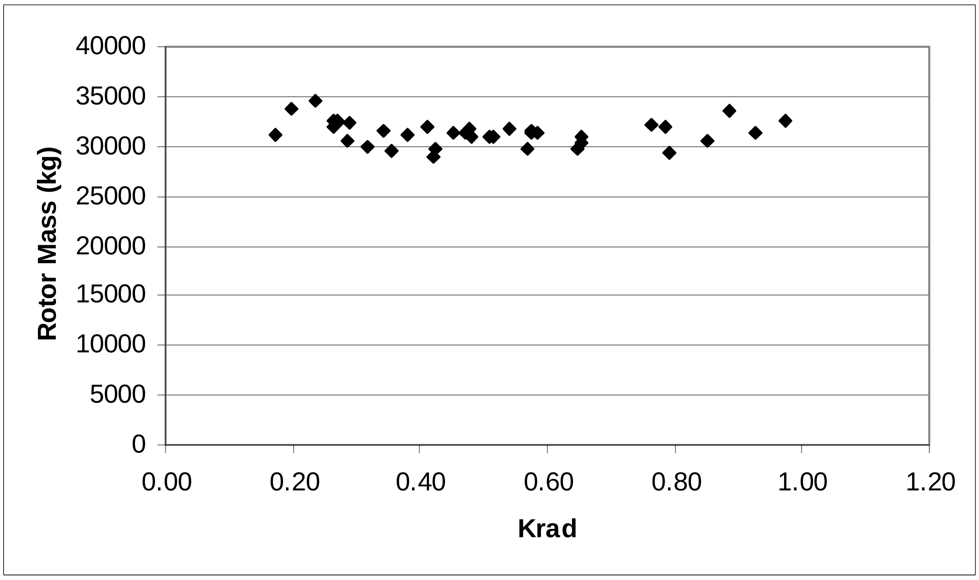

Fig. 6. Total rotor mass results for a 5MW generator rotor structure with arms for a range of K_{rad}

The optimal rotor with arms was found at a K_{rad} = 0.42 and a total mass = **28874** kg.
R=2.62m
l=2.2m
n=6
t=0.067m
b=0.69m
d=1.1m
t_w=0.011m

7. A variable airgap length

The constraint of the airgap length, *c* (and therefore radial deflection) being a constant proportion of the airgap radius can lead to an artificial optimisation. In this section the airgap length is also a variable – increasing the airgap length means that the structure can be lightweight but also means that the amount of magnetic material must be increased. A minimum airgap length of 0.002m is set and a maximum of 0.015m is included in the optimisation. It is assumed that the larger airgap lengths do not lead to extra flux linkage. The same airgap flux density of 0.85T is demanded from all the designs.

The best two designs give total rotor masses of **20415**kg (K_{rad}=0.82) and **20437**kg (K_{rad}=0.53).

K_{rad} = 0.82	K_{rad}=0.53
R=2.1m	R=2.42m
l=3.43m	l=2.58m
n=5	n=7
t=0.021m	t=0.023m
b=0.604m	b=0.59m
d=1.29m	d=1.32m
t_w=0.012m	t_w=0.010m
c=0.011m	c=0.014m
$c/2R$=0.0027	$c/2R$=0.0029

Figure 7 shows that there is a broad range for which the minimum mass can be found. What is more interesting is that the optimal airgap length, *c* is larger than normally seen in machine designs. Typically one would use $c/2R$=0.001, but Fig. 9 shows that lighter designs are found when the airgap is larger (and so the magnitude of the deflection is larger).

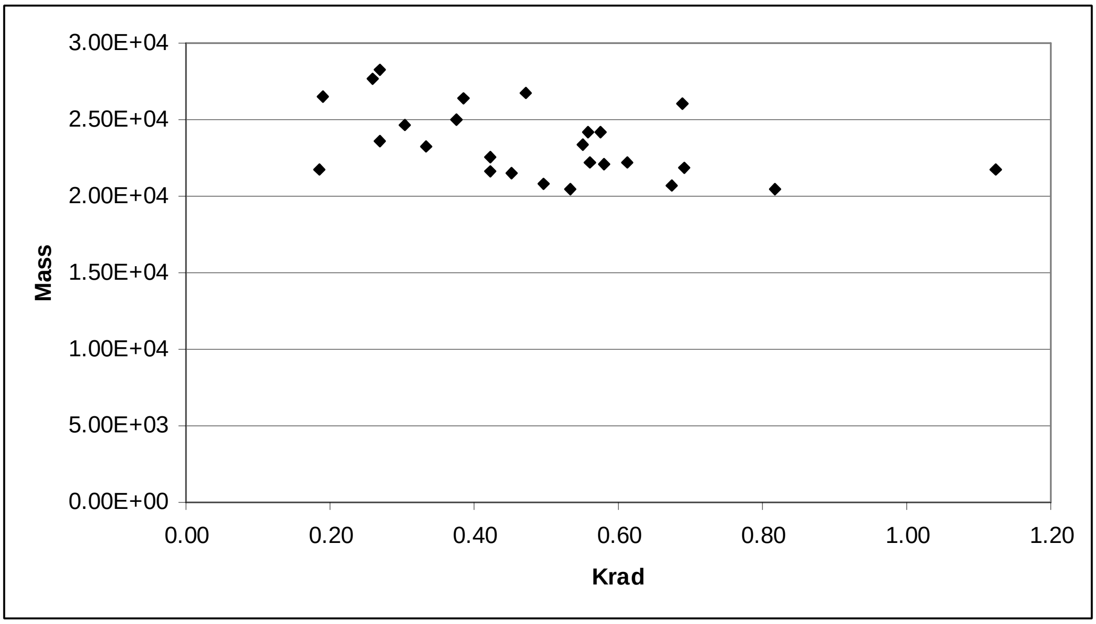

Fig. 7. Total rotor mass results for a 5MW generator rotor structure with arms – with a variable airgap length – for a range of K_{rad}

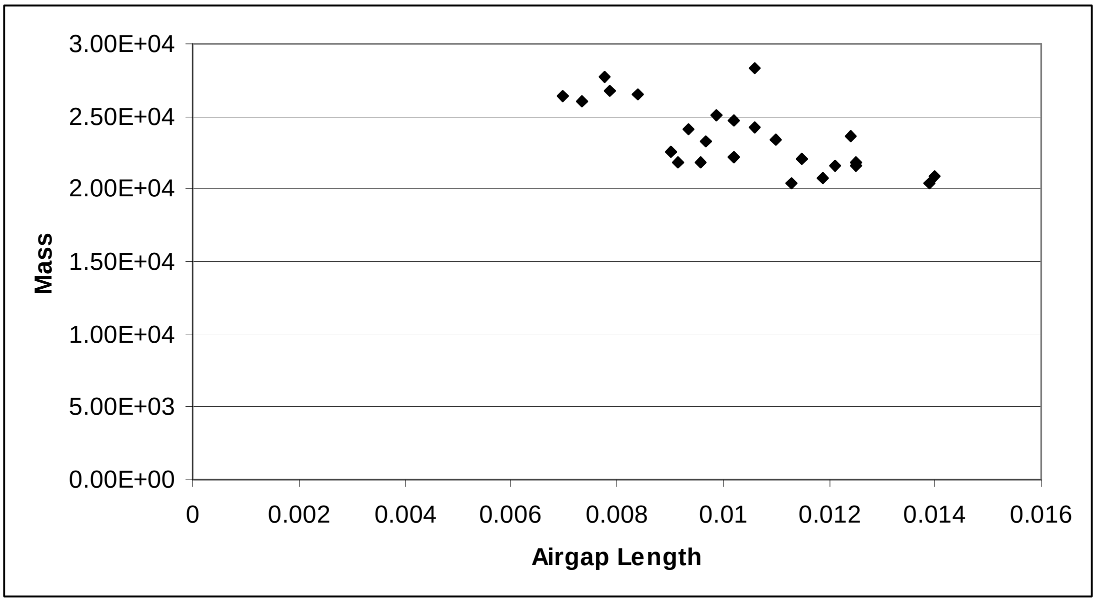

Fig. 8. Total rotor mass results (in kg) for a 5MW generator rotor structure with arms – with a variable airgap length – for a range of airgap lengths (in m)

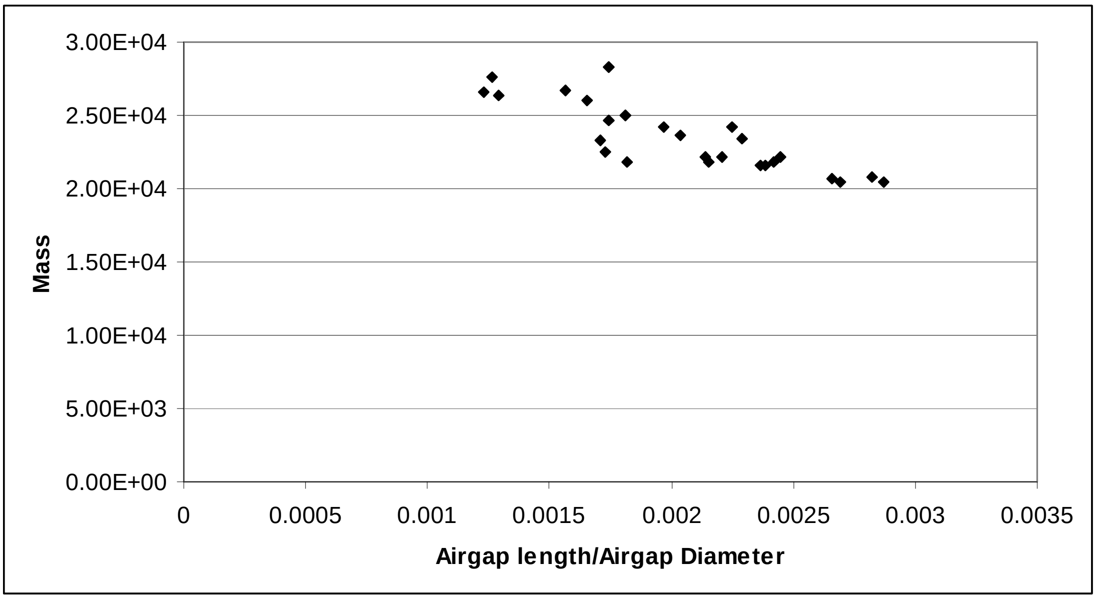

Fig. 9. Total rotor mass results (in kg) for a 5MW generator rotor structure with arms – with a variable airgap length – for a range of airgap length/airgap diameter ratio

8. A simple cost optimisation

A simple cost optimisation was carried out using the following specific costs: C_{PM}=35€/kg; $C_{steel,mag}$=25€/kg; $C_{steel,struct}$=10€/kg.

Figure 10 and 11 show that when optimising for costs a smaller K_{rad} and a smaller ratio of airgap length/airgap diameter are preferred. This is because the electromagnetically active materials (PMs and steel for the magnetic circuit) is more expensive and tends to be reduced at larger radii.

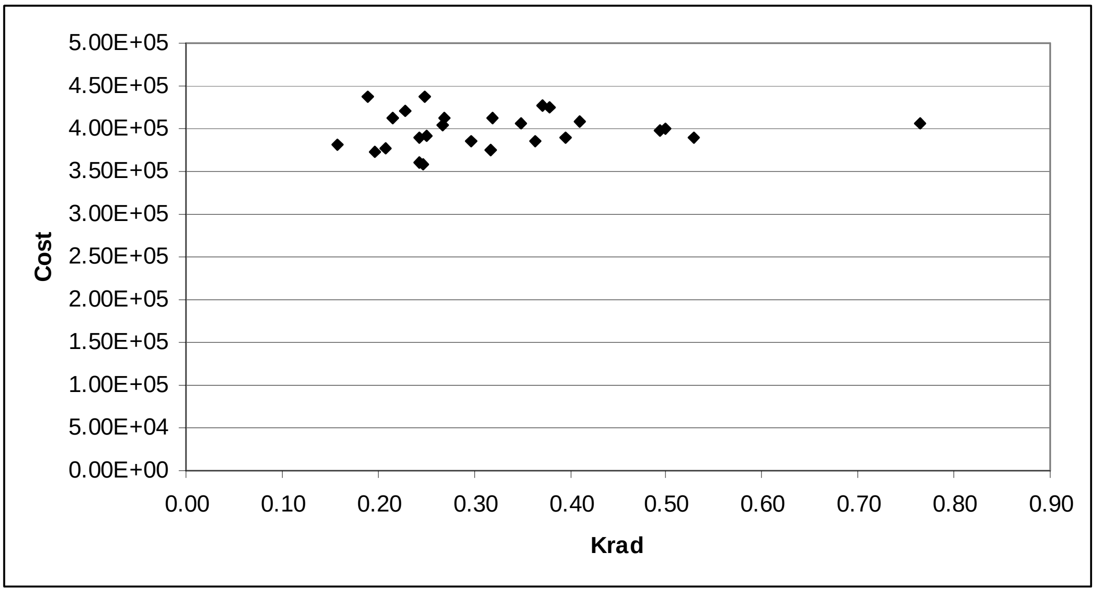

Fig. 10. Total rotor cost results (in €) for a 5MW generator rotor structure with arms for a range of K_{rad}

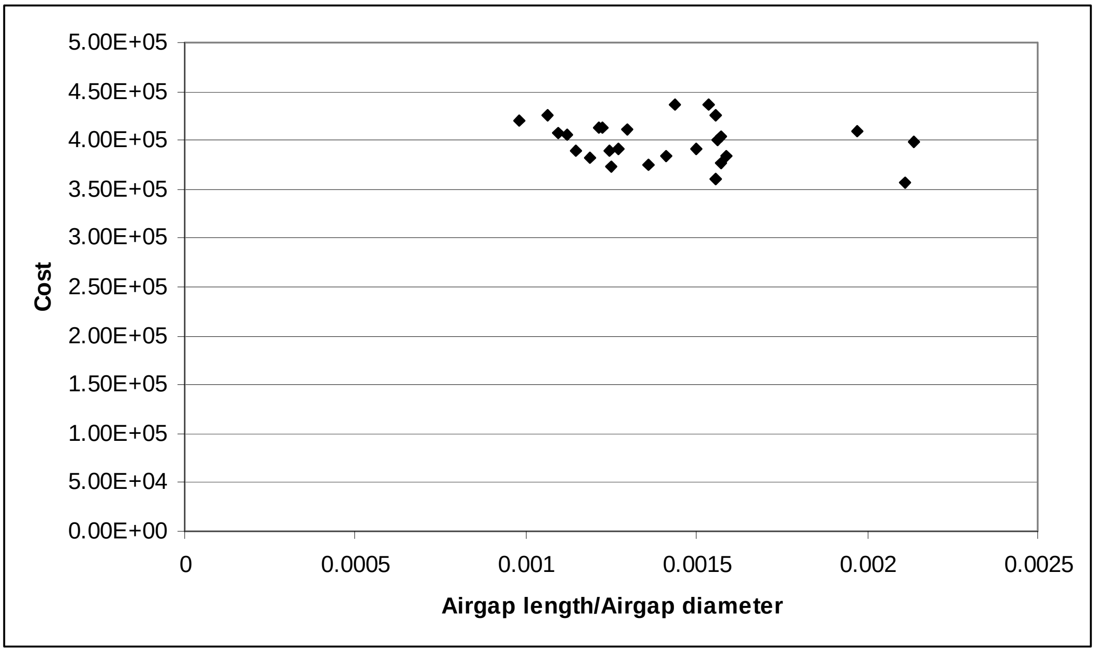

Fig. 11. Total rotor cost results (in €) for a 5MW generator rotor structure with arms for a range of airgap length/airgap diameter ratios

9. Discussion and Conclusions

Consecutive sections of this report have presented models and optimisations of increasing complexity, from looking at a rotor structure with a fixed aspect ratio all the way to the cost of the same structure (but with electromagnetically active material) with a variable aspect ratio and variable airgap length.

Along the way it has been seen that when only structural material is considered quite large aspect ratios are better for mass minimisation. Including the electromagnetically active material leads to smaller aspect ratios, particularly for cost minimisation. If a simplification of no significant leakage with increasing airgap length is assumed then it can be seen that lighter machines can be produced with a larger airgap length/airgap diameter ratio.

The methodologies outlined in this report allow the following "next steps" to be pursued:

- So far only a rotor with arms has been modelled – optimisation of a generator needs a simultaneous optimisation of the stator too.
- Rotor and stator structures with "arms" and "discs" from [2] will be modelled.
- The flux density will be a variable. Shear stress can be scaled by airgap flux density and the normal stress can be calculated accordingly $q = \frac{B_g^2}{2\mu_0}$.
- Sensitivity of the optimisation to relative material costs.
- Sensitivity of the optimisation to the deflection constraints.

10. References

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