BL3.199 Wake Modelling for intermediate and large wind farms

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Summary

Modern, very large wind farms require large-scale effects to be taken into account when evaluating wind turbine array efficiency – a requirement not met by contemporary engineering tools, and apparently not even by advanced steady-state CFD-like models. This paper presents an effort to fill the gap between academic models for infinitely large wind farms and present-day engineering models, which take into account only local flow characteristics. Thus, we describe a wind farm wake model which does not require any regularity of the wind farm layout, and which is based on first-principles in terms of volume- and momentum balance for relevant control volumes. It comprises a power-law evolution rule the for wake expansion downwind of each individual wind turbine, a set of equations describing the interaction of the wakes when overlapping and, on basis of that, a rule for evaluating the resulting mean speed deficit at some downwind turbine. For each wake, the evolution rule takes into account the extra-ordinary expansion caused by the local flow pattern of a surrounded turbine rotor. Downwind of a large wind farm, to evaluate the recovery of the wind flow, actual atmospheric boundary layer models will be needed.

1. INTRODUCTION

The increasing size of wind farms makes it necessary to revise the computational tools for wind farm efficiency to take large-scale effects into account. Contemporary engineering tools like WAsP [1, 2] with it's the Park-model [3, 4] do not do that. Thus, the Park-model neglects certain details in the wind-field very close to a turbine rotor, and very simplistic rules are applied to represent wake expansion and to evaluate the effect of wake overlapping. And further more advanced CFD-like models do not perform convincingly better [5].

The presented wake model is based on a work by Frandsen et al. [6] and aims at a sufficiently precise, yet simple and computationally fast method to calculate the wake effects in a wind farm. It is based on first-principle fluid-dynamics: global conservation equations for volume and momentum and - contrary to the model by Frandsen et al. [6] - it does not require any regularity of the wind farm layout, which is needed if the model should be used in connection with general wind resource software.

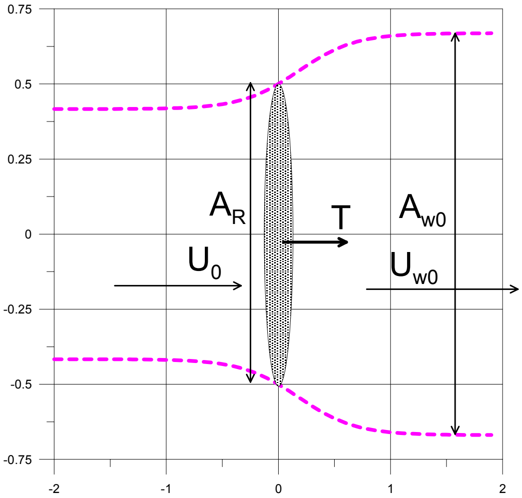
2. BASIC WAKE THEORY

There are two distinct wake-regions behind wind turbines: the near-field flow in the vicinity of a turbine rotor; and the region "far" downwind of the turbines, where the reduced wind speed in the wake(s) is needed as input calculating the production of the downwind turbine. These two regions must be treated separately.

2.1. Near-field

The near field may be described in terms of the following flow properties, as illustrated in fig.1.

- The turbine thrust T, expressed in terms of thrust coefficient C_T;
- The induction factor a, relating the wind speed U_{w0} immediately after the rotor to the free wind U_0 ;
- The expanded flow area immediately after the rotor, A_{w0}, related to the rotor area A_R through the expansion coefficient β, which in turn is related to a; and
- The total flow area expansion (from the stream-tube before to after the rotor) ΔA_T.



$$T = \tfrac{1}{2}\rho C_T U_0^2 \quad (1)$$

$$U_{w0} = (1-a)U_0 \quad (2) \qquad a = 1-\sqrt{1-C_T} \quad (3)$$

$$A_{w0} = \beta A_R \quad (4) \qquad \beta = \frac{1-\tfrac{1}{2}a}{1-a} \quad (5)$$

$$\Delta A_T = A_R a\beta \quad (6)$$

Figure 1. The near-field flow around a wind-turbine rotor.

2.2. *The far-field*

A cylindrical control volume is applied, aligned with the wind direction, containing all relevant turbines, and sufficiently wide, so that the speed deficit in the wind direction is vanishingly small at the cylindrical surface.

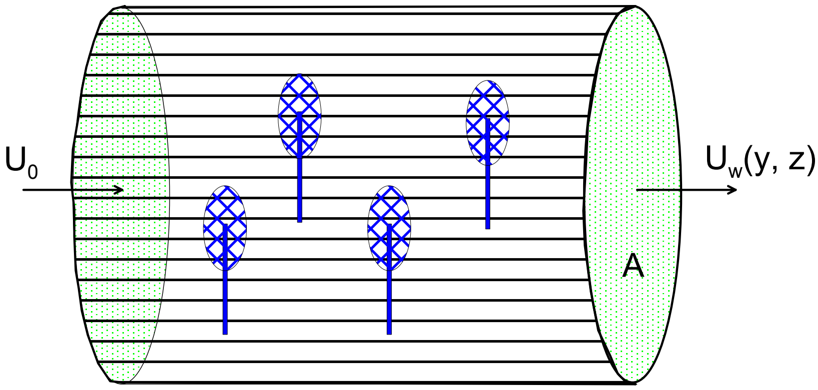


Figure 2. Cylindrical control volume around a set of turbines. In fact, the control volume should include a cut-off at the ground level, but for graphical reasons this has been left out.

Disregarding large-scale pressure gradients and shear forces at the control volume surface, volume and momentum balance equations for the control volume imply that the flow field $U_w(y,z)$ at some downwind position is related to the sum of turbine thrusts by

$$\frac{1}{\rho}\sum_i T_i = \iint_A U_w(y,z)\big(U_0 - U_w(y,z)\big)\,dydz \quad (7)$$

or expressed in terms of the relative speed deficit $\delta \equiv \dfrac{U_w - U_0}{U_0}$ (8):

$$\frac{1}{\rho U_0^2}\sum_i T_i = \iint_A \delta(y,z)\big(1-\delta(y,z)\big)\,dydz \quad (9)$$

Here the integration (y,z) is over the cross section of the exit area of the control volume.

2.3. Turbulence

Turbulence within a wake is expected to have an impact on the wake expansion rate, and also to vary in response to the wake expansion. However, an explicit treatment of turbulence intensity and it's impact on wake behavior has been left out to keep the present model sufficiently simple. The effect of turbulence will eventually be parameterized through the wake expansion variables.

3. WAKE EXPANSION

The single-wake expansion model is based on the work by Frandsen [6]. The wake is assumed to have a circular cross section. For a certain downwind cross section the true speed deficit profile is believed to be Gaussian-like. However, Frandsen showed that essentially no generality would be lost if one approximates the speed deficit profile by a so-called top-hat profile, and in the present model this approximation has been adopted:

$$U_0 - U_w(x,r) = \begin{Bmatrix} r \le \tfrac{1}{2} D_w(x) : \Delta U_w(x) \\ r > \tfrac{1}{2} D_w(x) : \qquad 0 \end{Bmatrix} \qquad (10)$$

x: downwind distance from the rotor from which the wake originates
r: radial distance from the wake centerline defined by the centre of the rotor

i.e. a profile with an x-depending speed deficit inside the wake boundary, and zero outside.

The wake expansion rule of the present model is based on the power-law expansion suggested by Frandsen, but extended to take into account the extra expansion occurring when passing a surrounded turbine rotor due to the near-field stream-line expansion occurring here, see Figure 1. Also, it has been taken into account that there are indications that the wake diameter tends to zero when extrapolated back to its originating rotor [7].

$$D_w(x) = D_R \left[\max\left(\beta^{k/2}, \alpha \frac{x}{D_R} \right) \right]^{1/k} \Psi \qquad (11)$$

Here α is of the order 1.0, k is between 2 and 3, and Ψ is a parameter or order unity which increases stepwise to account for the "extra expansion" due to surrounded turbine rotors. The wake cross sectional area is then – including a cut-off area at ground surface:

$$A_w(x) = \frac{\pi}{4}\left(D_w(x)\right)^2 - A_{Cut-off} \qquad (12)$$

The stepwise evolution of Ψ when passing a surrounded turbine "j" is governed by the following equation:

$$A_w(x_j, \Psi^{[j]}) - A_w(x_j, \Psi^{[j-1]}) = \Delta A_{T,j} \qquad (13)$$

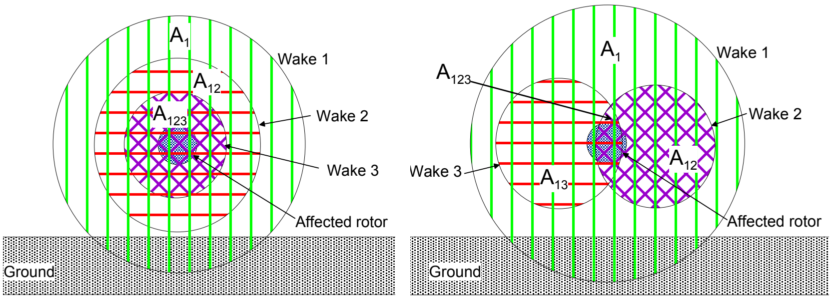
$$\Phi^{(j)} = 1 + \frac{\Delta A_{T,j}}{A_w(x_j, \Psi^{[j-1]})} = A_w(x_j, \Psi^{[j]}) / A_w(x_j, \Psi^{[j-1]}) \qquad (14)$$

Here ΔA_{Tj} is the stream-line area expansion around turbine "j" (eq. 6), and $\Phi^{(j)}$ denotes the corresponding wake area expansion ratio. Also, we introduce the relative increase in Ψ at turbine "j", $\Psi^{(j)}$, through

$$\Psi^{[j]} = \prod_{k=1}^{j} \Psi^{(k)} \qquad (15)$$

4. WAKE INTERACTION – Mosaic-tile model

The speed deficit distribution at some down-wind (turbine) position is approximated by a pattern of "mosaic tile" regions, each with one or more overlapping wakes, and - in line with the top-hat profile for single wakes – with a constant speed deficit. The principle is illustrated in figures 3A and 3B.



A. B.

Figure 3. Wake pattern examples.

From Eq.(9) one then get the following equation for the relative speed deficits for the tile regions, where the last summation is over all tile regions **J**:

$$\frac{1}{\rho U_0^2}\sum_{i=1}^{N_{turbines}} T_i = \sum_{\mathbf{J}}^{\mathbf{J}\in(Tile\,regions)} A_{\mathbf{J}}\delta_{\mathbf{J}}(1-\delta_{\mathbf{J}}) \tag{16}$$

It should be noted that the symbol **J** denotes a set of indices for the wakes making up the tile region in question; thus in figure 3.A, **J** would take the values [1] (wake 1 only), [1,2] (overlapping region of just wake 1 and 2) and [1,2,3] (overlapping region of wakes 1, 2 and 3**).**

This equation is in fact rather complicated but it may be solved for each of the relative tile speed deficits $\delta_{\mathbf{J}}$ as follows:

a. Apply the equation recursively to any subgroup of the *N* turbines,

b. Take the relative speed deficit for so-called *incomplete tiles* (tiles partly cut off by another wake) to be identical to those for the corresponding complete tiles, i.e. disregard other individual wakes than those defining the tile. E.g. in fig 3.A, wake 3 is disregarded when calculating $\delta_{[12]}$ and in fig.3.B wakes 2 and 3 are disregarded when calculating $\delta_{[1]}$.

c. Step b. involves back-correcting for the extra wake expansion, due to the turbines thus disregarded through the Φ-factors. This back-correction is accomplished by the use of effective tile areas $A^{+(\mathbf{J})}$, related to the treatment of each complete tile **J**.

For convenience we introduce the property $\varepsilon_{\mathbf{J}}$ related to $\delta_{\mathbf{J}}$ by

$$\varepsilon_{\mathbf{J}} \equiv \delta_{\mathbf{J}}(1-\delta_{\mathbf{J}}),\quad \delta = ½ - \sqrt{¼ - \varepsilon} = \frac{2\varepsilon}{1+\sqrt{1-4\varepsilon}} \tag{17}$$

Then for a certain tile region **J** the relative speed deficit is found as:

Tile with a single wake *i* (**J** = [*i*]) :

$$\varepsilon_{[i]} A^{+}_{[i]} = \frac{1}{\rho U_0^2} T_i \tag{18}$$

Tile with a set **J** of *q* overlapping wakes, *q* = Order(**J**)>1:

$$\varepsilon_{\mathbf{J}^{(q)}} A^{+}_{\mathbf{J}^{(q)}} = \frac{1}{\rho U_0^2}\sum_{i}^{i\in\mathbf{J}^{(q)}} T_i - \sum_{r=1}^{q-1}\sum_{\mathbf{K}^{(r)}}^{\mathbf{K}\subset\mathbf{J}^{(q)}} \varepsilon_{\mathbf{K}} A_{\mathbf{K}}^{+(\mathbf{J})} \tag{19}$$

The mean speed deficit $\delta^{(R)}$ over the rotor area $A^{(R)}$ of a downwind rotor is then found as

$$A^{(R)}\delta^{(R)} = \sum_{p=1}^{n}\sum_{\mathbf{J}^{(p)}} A^{(R)}_{\mathbf{J}^{(p)}}\delta_{\mathbf{J}^{(p)}} \tag{20}$$

where $A_{\mathbf{J}}^{(R)}$ denotes the part of the tile area overlapping with the rotor.

The effective tile areas A^{+} are related to the true tile areas by the following equations, where $A_{\mathbf{K}}^{*}$ denotes the gross area of tile **K**, i.e. the entire overlapping area of the wakes contained in the set **K**:

$$A_{\mathbf{K}^{(p)}}^{+(\mathbf{J}^{(q)})} = A_{\mathbf{K}^{(p)}}^{*} / \Phi_{\mathbf{K}^{(p)}}^{+(\mathbf{J}^{(q)})} - \sum_{r=p+1}^{q} \sum_{\mathbf{L}^{(r)}}^{\mathbf{K}\subset\mathbf{L}\subseteq\mathbf{J}} A_{\mathbf{L}}^{+(\mathbf{J})}, \ p < q \tag{21}$$

with

$$\Phi_{\mathbf{K}}^{+(\mathbf{J})} = \sum_{i\in\mathbf{K}} \left(w_i \prod_{l\notin\mathbf{J}}^{A_{\mathbf{K}\cup\{l\}}>0} \Phi_i^{(l)} \right) / \sum_{i\in\mathbf{K}} w_i, \quad w_i = 1 \big/ (A_i^{*}(x))^2 \tag{22}$$

5. TEST AGAINST WIND FARM DATA – STATUS.

The wake model has been tested against data from the Horns Rev offshore Wind Farm in the North Sea West of Esbjerg – see the illustration of figures 4 and 5.

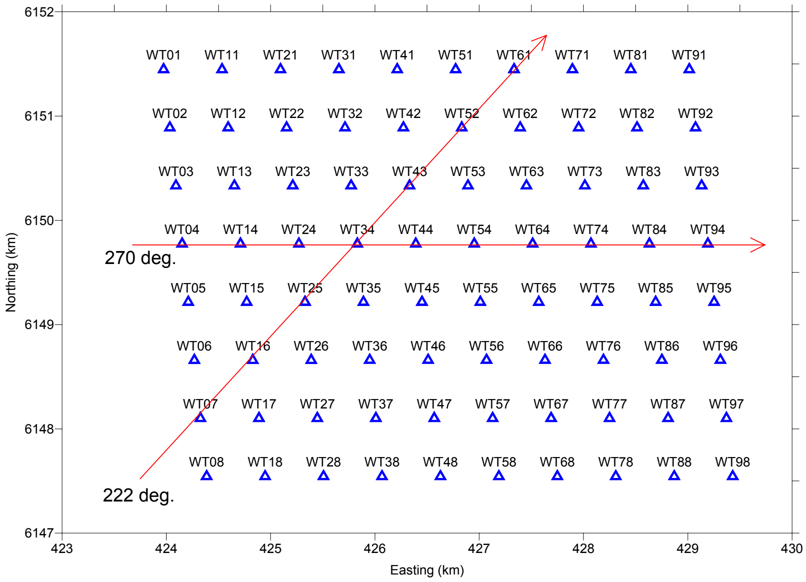


Figure 4. Horns Rev Wind Farm Layout.

- 80 Vestas 2MW turbines
- Rotor diameter: 80 m, Hub height: 70m.
- Spacing: about 7 rotor diameters.

The wind directions along the main rows and the diagonal rows are indicated by arrows. Wind data with these directions were used when comparing to model results.

Figure 5. Horns Rev Wind Farm. Aerial view.

Data for wind directions 270 ° (Westerly wind) and 222 ° (South westerly wind), in line with the main-row and the diagonal-row direction, respectively, were used as indicated in figure 4.

The results from a previous semi-linear version of the model have been compared to the wind farm data as shown in figure 6. Whereas the speed deficit level is in agreement with data the semi-linear model-version was not able to capture the details in the down-wind evolution of the speed deficit through the wind farm – also when varying the wake expansion parameter α.

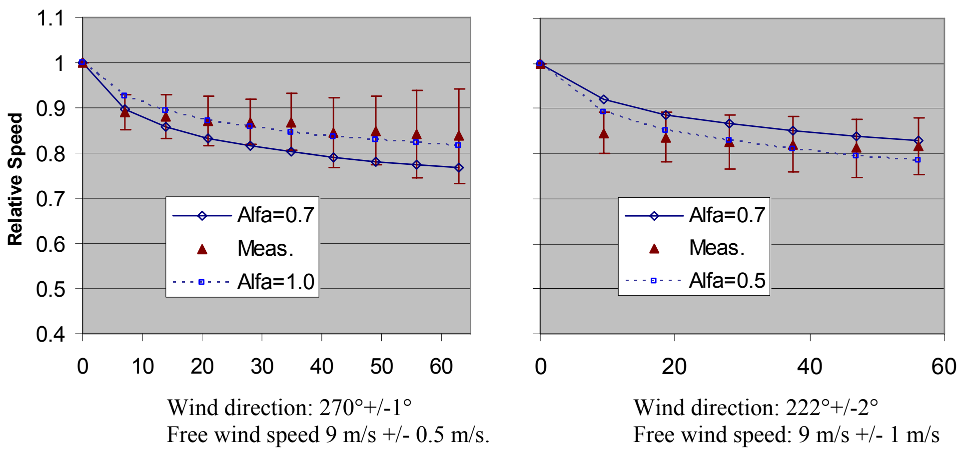


Figure 6. Comparison with results from semi-linear version of the model.

Results from the present model for an intermediate (8.5 m/s) and for a high wind speed (12 m/s) are presented in figures 7 and 8.

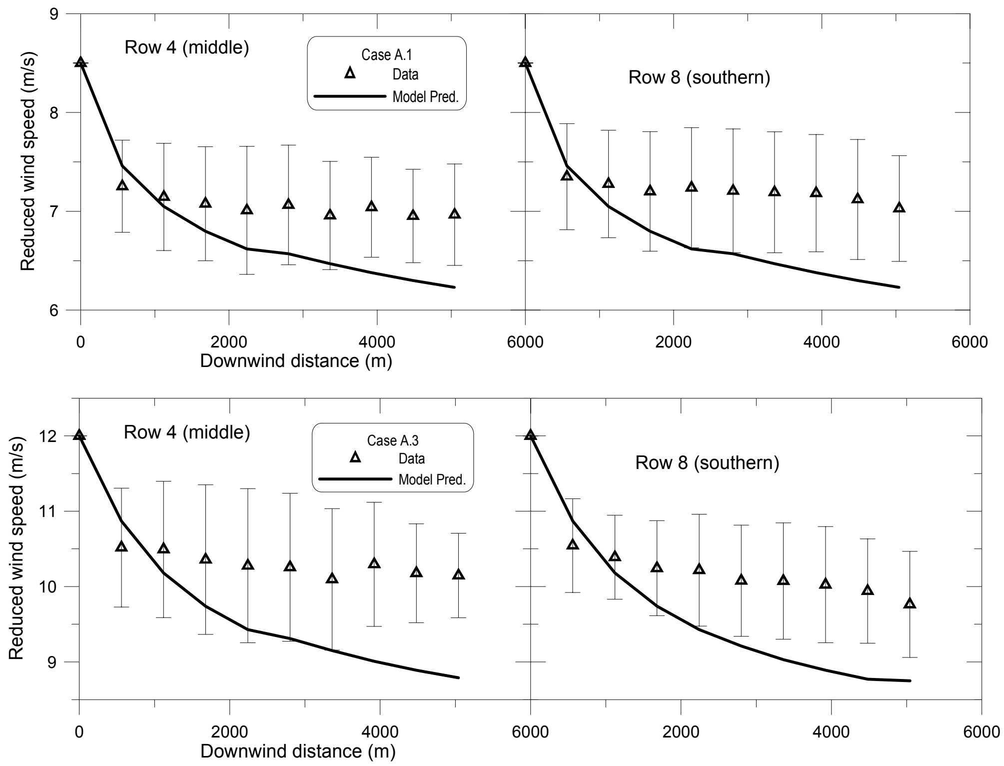


Figure 7. Model predictions at wind direction 270° +/-3° compared to data.
Free wind speed: 8.5 m/s +/- 0.5 m/s (top) and 12.0 m/s +/- 0.5 m/s (bottom).

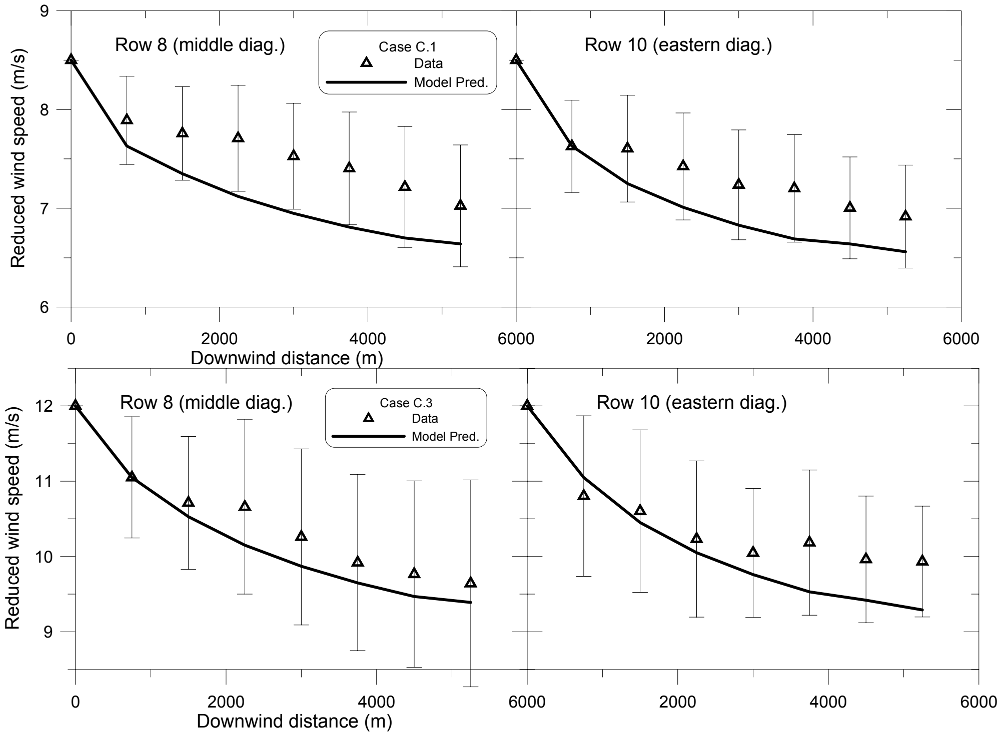


Figure 8. Model predictions at wind direction 222° +/-3° compared to data.
Free wind speed: 8.5 m/s +/- 0.5 m/s (top) and 12.0 m/s +/- 0.5 m/s (bottom).

This allows to see whether the comparison differ in any principal way due to the somewhat smaller turbine thrust at high wind speed. For both wind directions the comparison were performed for a central row of turbines (where wake overlapping from both sides may occur) and a row located at the boundary (with wake overlapping from only one side).

Clearly, the model is not able to predict the details in the observed downwind evolution of the speed deficit. The high wind speed case is not different in this respect from the intermediate wind speed. The discrepancy is especially evident for wind direction 270° where the predicted speed deficit continues to increase in contradiction to the measured data which levels out after a few downwind turbine positions. It is obvious to attribute this discrepancy to improper values of the wake parameters k and α in response to wind conditions and wake overlapping.

6. DISCUSSION

- The wind speed deficit at the first wake-effected turbine is generally predicted well;
- The new "mosaic tile" model - with the selected wake expansion parameters - seems to be able to yield a somewhat better prediction than the previous semi-linear model .
- The new model still does not reach a stationary reduced speed - as seen in the data.
- The wake expansion parameters may have to depend on the degree of local wake-interaction.

7. DOWNWIND REGION

The region downwind of large wind farms cannot be expected to be treatable by wake models since the wakes will typically have developed to a size where interaction with the atmospheric boundary layer starts to take place. Also, the turbulence level will be rather high and may lead to faster wind speed recovery than expected from single- or multiple wake considerations.
Instead, it is suggested that flow models for the atmospheric boundary layer may be used, possibly with the output from the present wake model serving as an inflow-condition.

8. CONCLUSION

- The new "mosaic tile" model seems promising, and is believed to be able to be developed to an efficient tool for estimating wake effects in large wind farms.
- The algorithm is in its present form is too time consuming; more efficient methods to calculate the wake tile areas must be developed.
- By comparison with more wind farm data, optimal values for the wake expansion parameters k and α must be found, including dependence on local wake interaction conditions.
- For the far-field region downwind of a wind farm the "mosaic tile" wake model may be combined with boundary layer models to treat this region separately.

9. REFERENCES

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